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# **TURBULENT FRICTION IN ECCENTRIC ANNULAR CONDUITS**

by

**James M. Robertson**

A report on

**"Effect of Inserted Pipe on Flow Capacity of Sewers"**

Conducted for

**Task 12 of the ASCE Combined Sewer Separation Project  
(Contract FWPCA No. 14-12-2)**

**DEPARTMENT OF THEORETICAL AND APPLIED MECHANICS  
UNIVERSITY OF ILLINOIS  
URBANA, ILLINOIS**

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## ABSTRACT

Following a general review of the analytical and experimental information on the friction loss encountered by fluids flowing in annular pipes, with particular regard to the influence of eccentricity of the inner member, experiments are described on an evaluation of the friction of water in a steel annular pipe of diameter ratios 5.8 and 3.2 in the Reynolds number range of  $10^5$  to  $10^6$ . It is found that, on a friction factor basis (in terms of the equivalent diameter as four times the hydraulic radius), whereas at the larger ratio the concentric position of the insert increases the friction only about 2 percent, its full eccentric location results in a decrease of some 15 per cent. On a discharge basis, however, for the same head loss in a given length, the flow capacity of the pipe line is decreased 12.7 per cent in the concentric situation but only 4.5 percent with full eccentricity. This decrease is not greatly different from the 3 percent reduction in area due to the inserted smaller pipe. An analysis is included showing that for the simple insert at full eccentricity the near-full-flow capacity of a sewer is little affected. The effects of hangers such as might be employed to support inserts in sewers is found to have an appreciable effect on the flow capacity of a full-flowing sewer.

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## I. INTRODUCTION

Fluid flow in a conduit of annular cross-section has been a subject of considerable interest to fluid dynamicists for over sixty years, as evidenced by the literature review and bibliography compiled in Section VI of this report. The function of this introductory section is to reference much of this wealth of data so that the present study may appear in its proper position and the results interpreted correctly in terms of the broad fund of knowledge on this subject. Before that objective is attempted, however, the reasons for interest in this flow problem are noted.

### The Annular Conduit Problem

Annular conduits are not uncommon for transporting fluids, but their most common occurrence is as heat-transfer devices in various applications. As a result many experimental studies have involved heat transfer as well. From a fundamental standpoint fluid dynamicists have been intrigued by the laminar-viscous flow prediction of appreciable effects of thin wires in tubes--thus a wire of one thousandth of the tube diameter theoretically will reduce its capacity by 14.5 percent. Furthermore, in the turbulent flow regime this conduit shape provides a severe test of the hydraulic-radius concept, commonly accepted as providing the means for correlating the frictional characteristics of various-shaped conduits. And, as will appear in the sections to follow, some appreciable deviations do appear.

Now, although the above hydraulic-radius question and a need to bring some order to the subject, may indeed justify the author's efforts in making a general review; this is hardly adequate justification for adding a few more experimental results to the already overwhelming mass of such data, unless enough cases are studied to clear up the vagaries apparent in the data for the concentric case. However, such was not achieved here; but, as noted in the following paragraph, there is a pressing need for an indication of eccentricity effects on the friction in turbulent flow and there is not a great deal of information on this point.

The specific motivation for the reported study stems from the plans of the Combined Sewer Separation Project study of the American Society of Civil Engineers to use pressure lines, through which the sewage is to be pumped, and when feasible to locate these pressure lines within extant combined sewers. It is important in such cases to know how much the storm water capacity of the sewer is reduced by the insert pipe. The storm big enough to fill the sewer to its capacity does not often occur. It is common practice to design the sewer for free surface flow with the runoff from a storm of 5 to 25 year mean

probable frequency of occurrence. Since, especially on the larger sizes, there may not be much freeboard (relatively), on rarer occasions the sewer will be full and may even flow with a surcharge of pressure. It is thus desirable to ascertain just what the effect of the inserted pipe on full flow will be. It is expected that the pipe will be mounted at or near the crown of extant sewers with special hangers, hence concern here has been with the insert pipe at maximum and near-maximum eccentricity.

#### Factors Affecting Annular Conduit Flow

The nature of the flow in annular conduits depends upon both fluid-dynamic and geometric variables. Of the latter type we have the diameter ratio

$$(1) \quad a = \frac{D_o}{D_i}$$

where  $D_o$  is the outer diameter (inside diameter of the pipe) and  $D_i$  the inner diameter (i.e. outside diameter of insert) and the eccentricity

$$(2) \quad \epsilon = \frac{2 \Delta c}{D_o - D_i}$$

where  $\Delta c$  is the offset of the center of the insert from that of the pipe. The range of this latter factor is from zero (concentric case) to unity, for all eccentricity. The range of  $a$  is from unity to infinity, although one expects effects of the insert to disappear long before the higher limit.

In the turbulent-flow regime, the relative roughness of the conduit wall must be considered, however definition of this important quantity involves fluid-dynamic considerations. So long as the insert and pipe are circular in cross-section, are straight and have their generators parallel  $a$  and  $\epsilon$  are the sole geometric parameters. That is of course, assuming the fully developed flow is involved, i. e. the conduit long enough for such to obtain.

Basic Fluid-Dynamic Consideration. It does not take much understanding of fluid frictional occurrences, and certainly resort to dimensional analysis is hardly needed, to assemble quite an array of factors influencing friction in annular conduit flow. Thus it is clear that the basic nature of the flow--laminar-viscous versus turbulent (and we leave compressibility out of the picture)--will engender quite different frictional occurrences, and furthermore, in the turbulent-flow regime, the friction must depend upon the Reynolds number  $IR$  just as it does in all other conduit flows. Also in turbulent flows the boundary roughness can have an important influence on friction, and it usually does. Rather than characterizing this via a measurable geometric

quantity -- which then cannot be found directly for any "natural roughness" -- it is common, if not conventional, to define the effective roughness height  $k$  via reference to an equivalent sand-grain roughness of Nikuradse (of course for the fully-rough flow condition). This would seem to include all the possibilities until one considers that the insert may be of different material or vintage -- and thus roughness -- than the pipe in which it is mounted.\* The actual surface of the inserted pipe is likely to be smoother than the sewer but its supports may cause it to appear rougher.

Within the geometric limitations on the problem, as noted above, and the assumption of a Newtonian incompressible (or effectively so) fluid, one expects the friction factor of the flow to depend upon the several factors noted below.

$$(3) \quad f = \text{func} (a, \epsilon, IR, k/D, k_o/k_i)$$

where that last factor is the ratio of the  $k$ 's for the two surfaces. The exact definition for  $D$ , and thus  $IR$ , is elucidated in the next paragraph. Before passing on, one pauses to gasp at the complexity which has appeared in this seemingly simple conduit-flow problem. In the usual conduit, the friction factor depends upon  $IR$  and  $k/D$ , but now our insert has introduced three more factors.

In view of the obvious complexities of the flow problem, care must be taken in defining the basic reference quantities. As is common in most fluid-mechanical considerations of turbulent conduit flow, the basic length characterizing the size of the conduit is obtained via the hydraulic radius concept. The hydraulic radius is thus defined as the ratio of flow area to its "wetted" perimeter. Since for a pipe (conduit of circular cross section) this is one-fourth the diameter, an equivalent diameter is thus defined as four times the hydraulic radius. For the annular conduit, we thus have

$$(4) \quad D_{eq} = 4 \frac{(\pi/4) (D_o^2 - D_i^2)}{\pi(D_o + D_i)} = D_o - D_i$$

The hydraulic radius concept of turbulent flows is that substitution of  $D_{eq}$  in place of the pipe diameter, into the conventional relations for friction evaluation in pipe flow, will properly account for the effects of the difference in shape of a non-circular conduit. As will appear, this does not work that simply but the concept does provide a useful starting

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\* Except for the study of Nicol and Medwell (1964), who investigated a smooth pipe with an artificially roughened core, this possibility has not been given any systematic consideration.

point and correlation basis for considering non-circular conduits.

With the above definition for the equivalent diameter, the friction factor considered in this report is then defined via the Darcy-Weisbach pressure-drop relation

$$(5a) \quad \frac{\Delta p}{\gamma} = \Delta h = f \frac{L}{D_{eq}} \frac{U^2}{2g}$$

where  $\Delta p$  and  $\Delta h$  are the pressure and head drops in conduit length  $L$  for a mean flow velocity of  $U$ ;  $\gamma$  is the fluid specific weight and  $g$  the gravitational constant. The friction factor is then defined as

$$(5b) \quad f = \frac{\Delta h}{L} \frac{D_{eq}}{U^2/2g}$$

The reference Reynolds number is defined in terms of  $D_{eq}$  and the throughflow velocity, thus

$$(6) \quad R = D_{eq} U / \nu$$

where  $\nu$  is the fluid kinematic viscosity, as specified by the fluid temperature.

**Considerations of the Pipe Line.** Certain features of the conduit as an engineering structure can still significantly affect the flow occurrences. The conduit is not always so long that one need only consider fully-developed flow and the insert does not automatically maintain its prescribed concentricity or eccentricity. To some extent these considerations transcend the present report and study, but not entirely. Thus in any pipe, or conduit, friction experiment one must have an adequate length (generally thirty to fifty diameters) of straight run for the flow to develop in ahead of the test reach. It is not certain that some of the earlier experimentors had an adequate approach length and also one has the question as to whether the requisite length for an annular conduit (perhaps eccentric also) is the same number of equivalent diameters as has been found appropriate for the simple circular pipe. There is some indication in the literature that a longer length is needed.

The question as to whether the annular pipe has the geometry that one assigns to it at all stations along the test length obviously has a negative answer when one recalls that both the pipe and insert are flexible members. In the common horizontal lay of the conduit, the insert is more apt to suffer from excessive deflection. The avoidance of this is not merely a matter of using more and stiffer supports, since these obstruct and modify the flow. Some researchers have attempted to avoid this problem by using a

vertical alignment of their test pipe with the insert held in place by tension applied through end fixtures. This is not necessarily the perfect solution since one then tends to keep the test length rather short.

Some of the later researchers into annular conduit flow have discounted some earlier experimental results on the basis that too short an approach length was employed or that the insert was not adequately maintained in its axial position.

#### Review of Information on Flow in Eccentric Annular Conduits

As already noted, annular conduit flow has been under study by fluid dynamicists for many years and, as part of the present study, an attempt has been made to review the extensive literature which has resulted. As will appear, the information which has been gleaned is most useful in establishing or suggesting the mode of variation in the friction with the fluid dynamic parameters  $a$  and  $\epsilon$ . Both laminar and turbulent flow studies have been considered; the need for reference to laminar occurrences may be questioned but, in view of the incomplete understanding of turbulent flow occurrences currently obtainable, the laminar-viscous variations are suggestive of the type of effects to be expected with turbulence. Of course one must ultimately have some turbulent flow results to establish the occurrences and such is the particular object of the studies reported here.

Section VI summarizes the review made in the form of notes on the summary discussions appearing in several standard reference books and one summary paper together with a rather complete bibliography, listed chronologically, of published papers on the subject. All but three of the some forty papers listed have been studied during the course of this review. A brief summary of the pertinent indications are presented in the following discussions. Reference to the bibliographic items are via the author's name and indication of the date of publication in parenthesis.

Laminar-Viscous Flow Predictions. The solution for the laminar flow of a viscous fluid through the concentric annular conduit is relatively easily obtained from the Navier-Stokes equations, as indicated in some of our advanced texts on fluid mechanics.\* The literature review of Section VI of this report notes a number of authors, starting with Becker in 1907, who have found this solution. Following Richter, it is most convenient to relate the friction expression to that for the circular conduit, thus

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\* Thus Lamb's Hydrodynamics, p. 587.

$$\begin{aligned} f_{IR} &= 64 && \text{for the circular conduit (pipe)} \\ &= F 64 && \text{for the concentric annular conduit} \end{aligned}$$

where  $F = \text{func}(a)$ . Thus we have

$$(7) \quad F = \frac{f_{\text{ann}}}{f_{\text{cir}}}$$

as a definition for use in laminar or turbulent flow. Now in terms of this factor the laminar flow prediction for the concentric case is

$$(8) \quad F = \frac{(a - 1)^2}{1 + a^2 - \frac{a^2 - 1}{\ln a}}$$

where  $a = D_o/D_i$ . In the limiting cases of  $a = \infty$  and 1 this properly reduces to the 1.00 and 1.50 values characteristic of the circular conduit and wide channel.

In view of the success with which solutions (when achievable) of the Navier-Stokes have predicted other laminar-viscous flows of Newtonian fluids, one is apt to regard the prediction of Eq. 8 as sacrosanct. Admittedly this blind faith is put to a severe test when one considers a fine wire running down the center of a pipe. The predicted effect of this very small insert in reducing the flow, or increasing the friction at a given flow, seems most unreasonable. Lea and Tadros in 1931 were not too successful in verifying the predicted effects for small insert diameters and attempted to explain the deviation from theory via slip. Later Piercy, Hooper and Winney (1933) found agreement at an  $a$  of 49.5 to within 0.8 percent on the friction; at larger  $a$  values they were unable to obtain agreement with theory and found that their results varied with each resetting of the core in the pipe. They suggested that a longer approach length or considerably reduced inlet disturbances are needed to properly study the flow with insert cores of smaller size.

The effect of eccentricity of the insert on the frictional effect is known in general, but has not been as thoroughly studied. Thus as early as 1907 Becker (see indication in the summary of Schiller's review, Section VI of this report) indicated the effect for fine clearances, i.e.,  $a$  close to unity. In that limiting case, full eccentricity leads to a flow increase to 2.5 times, or a friction factor reduction (at the same Reynolds number) to forty percent of, the concentric value. This is a tremendous effect and certainly helps explain some of the anomalies found experimentally. There seems to have been no

experimental verification of this prediction however. The relation for small clearances ( $a \approx 1$ ) is

$$(9) \quad \frac{f_{ecc}}{f_{conc}} = \frac{1}{1 + 1.5 \epsilon^2}$$

as derived in Appendix A.

The general case of viscous flow in eccentric annuli, i.e., for  $a$  not restricted to near-unity values, has been attacked analytically by Caldwell (1930), by Piercy, Hooper and Winney (1933), by Heyda (1954) and by Redberger and Charles (1962). Caldwell solved the Poisson equation which results from the Navier-Stokes equation via a transformation from the bipolar coordinate system characterizing the geometry. His solution for the velocity profile, discharge relation and a friction coefficient came out in series form which he evaluated (for the friction) only in the case of the  $a$  value studies experimentally,  $a = 1.183$ . Piercy et al. achieved their solution via reference to the analogous torsion problem; the effect of eccentricity was evaluated for  $a = D_0/D_1$  values of 100, 2.0, 1.67, 1.43 and 1.0. The general trend of change in discharge with eccentricity is similar to that of Caldwell and, in fact, they state that the discharge relation agrees with his (although it appears in different form). As appears in Fig. 2, the predictions of Piercy et al. of the eccentricity effects are somewhat at variance with those of the other investigators and some rather strange variations for the  $a = 2.0$  case, particularly near full eccentricity, resulted. It therefore is presumed that their indications involved some errors of calculation. Heyda (1959) found the velocity profiles analytically via a Green's function solution; the results appeared in series form and were not integrated to obtain relations between the flow and pressure gradient.

An entirely different mode of solution for the Poisson equation was offered by Redberger and Charles (1962) as a numerical iterative solution of the corresponding finite-difference equation. The solution was in a rectangular mesh, of 512 points, obtained by transformation from the bipolar coordinate system describing the geometry. Solution was made with a digital computer and agreement with the analytical result of Heyda is noted for the case of  $a = 4$ . Integration of the velocity profiles for  $a$  values of 1.11, 1.33, 2.0, 4.0 and 10 led to indications of the flow rate changes as a function of  $a$  and  $\epsilon$ . The general trends are as found by previous investigators and the close agreement (see Fig. 2) with the numerical value worked out by Caldwell strongly suggests that it is the Piercy results which are somewhat in error.

Turbulent-Flow Observations. In the absence of any exact theory of turbulent conduit flow, one is generally forced to resort to empirical, or at best semiempirical, approaches to friction prediction. The annular conduit flow case is no exception to this general rule, however the work of Deissler and Taylor (1955) and Macagno and McDougal (1966) furnish encouraging exceptions. In view of the many variables which have been noted as affecting turbulent annular conduit flow and the difficulty of controlling some of these, it is not surprising that a large amount of variability appears when one attempts empirical correlation of the friction data. Even though fewer variables are involved in the laminar-viscous case, quite probably almost as much difficulty would appear in a correlation of such experimental data. Some success in an empirical correlation in both the laminar and turbulent cases was indicated by Davis (1943) and this has been accepted by Knudsen and Katz in their book. However the present study indicates disagreement with this correlation in the turbulent case. For the laminar flow case, one fortunately is always able to check the appropriateness of an experimental result via reference to the theoretical solution; thus the effect of a small concentric insert wire as verified by Lea and Tadros and by Piercy, Hooper and Winney. Such reference to higher authority is not available to the experimenter in turbulent flows.

Most experiments on the friction in annular flow have been limited to the concentric annulus, or rather what was hoped to be the concentric orientation. In all probability such may not always have been the true situation. Of the many papers listed in the bibliography, for which friction data was obtained in the turbulent regime of flow, only four involved the eccentric case. These are the study of Caldwell (1930) at  $a = 1.183$  and two eccentricities besides that of  $\epsilon = 1$  and 0, that of Stein, et al., (1954) only for the concentric case and that of full eccentricity at  $a = 1.307$ , the  $a \rightarrow 1$  study of Tao and Donovan (1955) and the recent work of Jonsson and Sparrow (1966) with three diameter ratios at four eccentricities. In view of the near unity values of  $a$  involved in all except the last of these, one can question the applicability of the indications to conduits of moderate diameter ratio.

Of the twenty-two sets of friction studies in presumably concentric annular conduits some were not considered too seriously here as they were at either low values of  $a$  or the Reynolds numbers were not far above those of transition to turbulent flow. One difficulty of comparison on the  $F$  basis, or any other for that matter, is establishing the appropriate pipe reference line. Where the surfaces are smooth this is generally not difficult--Lonsdale and Lea and Tadros found the friction to vary as the 1.72 and 1.73

power of the velocity, not far from the Blasius smooth pipe value of 1.75--but in some cases the nature of the surface is not noted and no reference pipe tests are available. In other cases, as the 6 in. pipe studies of Frazier (1948) and Owen (1951), the friction factor in the reference pipe does not appear to vary with Reynolds number in the same fashion as found with the annular conduit; some authors note their  $f$  values to vary with  $IR$  to the 0.125, 0.20, 0.21, 0.255, or 0.28 exponent. In such a case  $F$  appears to vary with  $IR$  sometimes even from below to above unity; such may indeed be the case or it may merely be an experimental vagary. As another anomaly experimental results for air and for water did not agree in the studies of Becker and Lonsdale. In at least some cases, it has been suggested that the test arrangement did not include an adequate length of approach conduit to develop the flow before the friction measurements were taken; thus Piercy says that Atherton's tests were deficient in this regard, as employing only fifteen diameters of approach, and Owen similarly objects to the Kratz results. Whether one can discard one set of data which seems too high and another which seems too low, both for the same failing of test setup, seems a bit tenuous. However the remaining data available in the  $a$  range between unity and 1.6 do seem to be consistent (see Fig. 1). The scatter in the  $F$  indications at higher diameter ratios is not as easily explained.

Some indications of the nature of the various test data, which have been considered in the correlation of diameter-ratio effects on the factor  $F$  in turbulent flow, are presented in Table I. The nature of the surfaces involved was not always clearly identified and in other cases it is not certain that the test Reynolds numbers were far enough removed from the transitional range. In general the results considered and presented in Figs. 1, 2 and 3 are those found at the higher Reynolds numbers of the particular study. In a few cases the range of  $F$  values are shown in Fig. 1.

As a final item of consideration from the literature on turbulent flow in annuli, there are the analyses of Deissler and Taylor (1955) and of Macagno and McDougal (1966). The first approach is an extension of the semi-empirical analysis by which the turbulent velocity distribution has been developed for symmetrical flows--thus the circular pipe and wide rectangular channel. Deissler assumes that along lines normal to the isovels the velocity variation with distance from the wall is the same as it is in the simple symmetrical flow despite the curvature of these lines for the eccentrically oriented annulus. Since the isovels are not known on an a priori basis an iterative approach is employed. For wall distances outside the viscous sublayer (and buffer zone) Deissler employed von Karman's similarity relation for the profile with the value of his

Table 1: List of Turbulent Annular Friction Tests Considered

| Researcher                 | Fluid              | Reynolds No's                        | Conduit              |                              |
|----------------------------|--------------------|--------------------------------------|----------------------|------------------------------|
|                            |                    |                                      | Size                 | Surface                      |
| Becker (1907)              | Water              | 3000 to 7600                         | 0.87 in.             | smooth                       |
| Winkel (1923)              | Water              | up to 6400                           | 1.2 in.              | smooth                       |
| Lonsdale (1923)            | Water              | up to abt. $4 \times 10^4$           | 0.75, 1.0<br>1.6 in. | smooth brass                 |
| Atherton (1926)            | Water,<br>Air, Oil | $10^4$ to $10^5$                     | 2.3 in.              |                              |
| Schneckenberg (1929/1931)  | Water              | up to $10^4$                         | 3.2 in.              | smooth                       |
| Caldwell (1930)            | Water              | up to $4 \times 10^3$                | 0.74 in.             | smooth brass                 |
| Kratz et al.(1931)         | Brine              | $3 \times 10^3$ to $3 \times 10^4$   | 2.0 in.              | wrought iron                 |
| Lorenz (1932)              | Water              | $10^4$ to $2 \times 10^5$            | 7.5 in.              | smooth                       |
| Piercy et al.(1933)        | Air                | up to abt $3 \times 10^3$            | 0.51 in.             | smooth brass                 |
| Carpenter et al.(1946)     | Water              | up to 9600                           | 0.83 in.             | copper, smooth               |
| Frazier*(1948)             | Water              | $3 \times 10^4$ to $4 \times 10^5$   | 6 in.                | used steel<br>(insert steel) |
| Owen (1951)                | Water              | $10^4$ to $2 \times 10^5$            | 2 in.                | smooth brass                 |
| Rothfus et al.(1950)       | Air                | $4 \times 10^3$ to $2 \times 10^4$   | 3.1 in.              | smooth brass                 |
| Stein et al.(1954)         | Water              | $10^4$ to $10^5$                     | 1.1 in.              | smooth aluminum              |
| Tao and Donovan (1955)     | Water              | up to $3 \times 10^4$                | 3.0 in.              |                              |
| Barrow (1955)              | Air                | $1.4 \times 10^4$ to $5 \times 10^4$ | 3.5 in.              | smooth                       |
| Brighton and Jones (1964)  | Air                | $3 \times 10^4$ to $3 \times 10^5$   | 8 in.                | smooth aluminum              |
|                            | Water              | $4 \times 10^3$ to $1.6 \times 10^4$ | 0.62 in.             | plastic tube with<br>wires   |
| Jonsson and Sparrow (1966) | Air                | $2 \times 10^4$ to $1.6 \times 10^4$ | 4 in.                |                              |
| Quarmby (1967)             | Air                | $6 \times 10^3$ to $9 \times 10^4$   | 2.9 in.              | smooth brass                 |
| Okiishi and Serovy (1968)  | Air                | $8 \times 10^4$ to $1.6 \times 10^4$ | 4 in.                | smooth brass<br>and copper   |

\*Given by Owen

parameter taken as 0.36. On this basis the velocity distribution was found in an annulus of  $a = 3.5$  at eccentricities  $\epsilon$  of 0, 0.6, 0.8 and 1.0. From these results the gross frictional and heat transfer effects were predicted.

The above-noted analysis for turbulent flows in annuli is obviously merely an estimate of the true occurrences, but it is thought to be quite a good one. In all probability it underestimates the friction for appreciable eccentricities, since in such cases the

turbulent velocity field is not symmetrical and secondary currents tend to develop. The effect of these is probably not large so that this analysis promises considerable aid in the evaluation of the various factors affecting the friction of annuli, these would be most difficult to ferret out with any certainty by the experimental approach. It should thus be possible to predict the mode of variation in friction factor with diameter ratio, eccentricity, and roughness--even for the case of the two surfaces of different roughness. It is hoped to carry out such analyses with the aid of a digital computer as a second phase of this study of turbulent friction in annuli.

#### Variation in Turbulent Friction Factor with Diameter Ratio and Eccentricity

The information on friction in annular flow, as just reviewed, permits one to make a reasonable estimate of the effects of diameter ratio and eccentricity for the turbulent case, on the assumption that the roughness of the surfaces had a negligible effect and that the relative occurrences are independent of Reynolds number. These assumptions can hardly be universally valid, but the pipes surfaces studied did not differ greatly in roughness and the Reynolds number ranges of the experiments were generally much the same. It is well-nigh impossible to separate out all the effects via experiment, as is evidenced by considering the roughness effect. For reference, one needs the friction factor vs. Reynolds number curve for a pipe of the same roughness and this is most easily found by tests on the pipe before the inserts are added. This would appear to be the way to conduct the study until it is realized that addition of the insert--hopefully of the same roughness height--has changed the effective diameter so that the relative roughness is now different although the surface is not.

As a basis for evaluating the effects of diameter ratio and eccentricity, it has been assumed that their effect on the friction factor may be considered separately and then combined as a product. Thus

$$(10) \quad f = f_{\text{pipe}} F E = f_{\text{pipe}} F [1 - \Delta (1 - E_1)]$$

where  $F$ , according to Eq. 7, is the ratio of the friction factor of a concentric annular to that of a pipe (circular conduit) under similar conditions (Reynolds number and relative roughness). The factor  $E$  is the eccentricity correction, defined as the ratio of the friction factor of the eccentric annulus to that of the concentric one. The nature of the factors  $F$  and  $E$  (as well as  $\Delta$ ) are discussed separately below.

Turbulent Friction of Concentric Annulus. The various sources of information on the friction factor of the concentric case have already been noted (see Table I), and some

attempt has been made to assess their reliability. But as hinted, in the end one has to be rather arbitrary and fit the most probable trend line through the regions where the data points appear most dense.

The experimental data on turbulent friction in concentric annuli is presented in Fig. 1. The data of Kratz et al. and of Atherton have already been discussed and some doubt thrown on them in terms of inadequate approach lengths. In view of the fact that the results appear quite low, the Kratz experiments may have suffered from a non-concentric mounting. In any case they appear abnormally low, while those of Atherton are much higher than any others, at least for the lowest  $a$  value. The results of Piercy et al. at  $a$  of two and above, of Rothfus at about six as well as the 6 in. pipe result of Frazier and Owen at  $a = 1.76$  all appear rather high. Other values by some of these researchers appear in reasonable agreement with the general consensus. It should be noted, however, that the Frazier and Owen result at 1.76 is subject to considerable uncertainty, due to the definition of the friction in reference pipe line and variations with Reynolds number, so that possible values for  $F$  range between about 1.1 and 1.4. The associated tests at  $a = 2.55$  seem to involve much less uncertainty.

The suggested trend line in Fig. 1 for the variation in  $F$  with  $a$  is reasonably well justified by the majority of the data, although more definitive data at  $a = 2$  is needed. The value suggested at  $a = 1$  is based on the friction factor relation for a wide rectangular channel\*

$$(11) \quad \frac{1}{\sqrt{f}} = 2.03 \log \left( \frac{D_{eq} U}{\nu} \sqrt{f} \right) - 0.47$$

as obtained from integration of the universal logarithmic-type velocity profile. The difference between the above relation and the corresponding one for the circular conduit

$$(12) \quad \frac{1}{\sqrt{f}} = 2 \log \left( \frac{DU}{\nu} \sqrt{f} \right) - 0.80$$

involves the shape of the conduit and suggests an  $F$  value at  $a = 1$  of 0.895 for  $Re = 6 \times 10^4$  and 0.913 for  $8 \times 10^5$ . The mean of these two is taken in Fig. 1. It does seem unexpected that this  $F$  value should be less than unity, in view of the laminar one being appreciably larger than unity and the turbulent-flow indications, for most  $a$  values, of  $F$  as somewhat greater than one; but further consideration, in terms of the amount of flow area associated with a given portion of the wetted perimeter does suggest that the very wide rectangular conduit should be more efficient than the circular one. A final check on this indication is the rather good agreement with the resultant trend line, up to  $a$  of about 1.5, of the

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\* See Rouse, Elementary Mechanics of Fluids, p. 214.

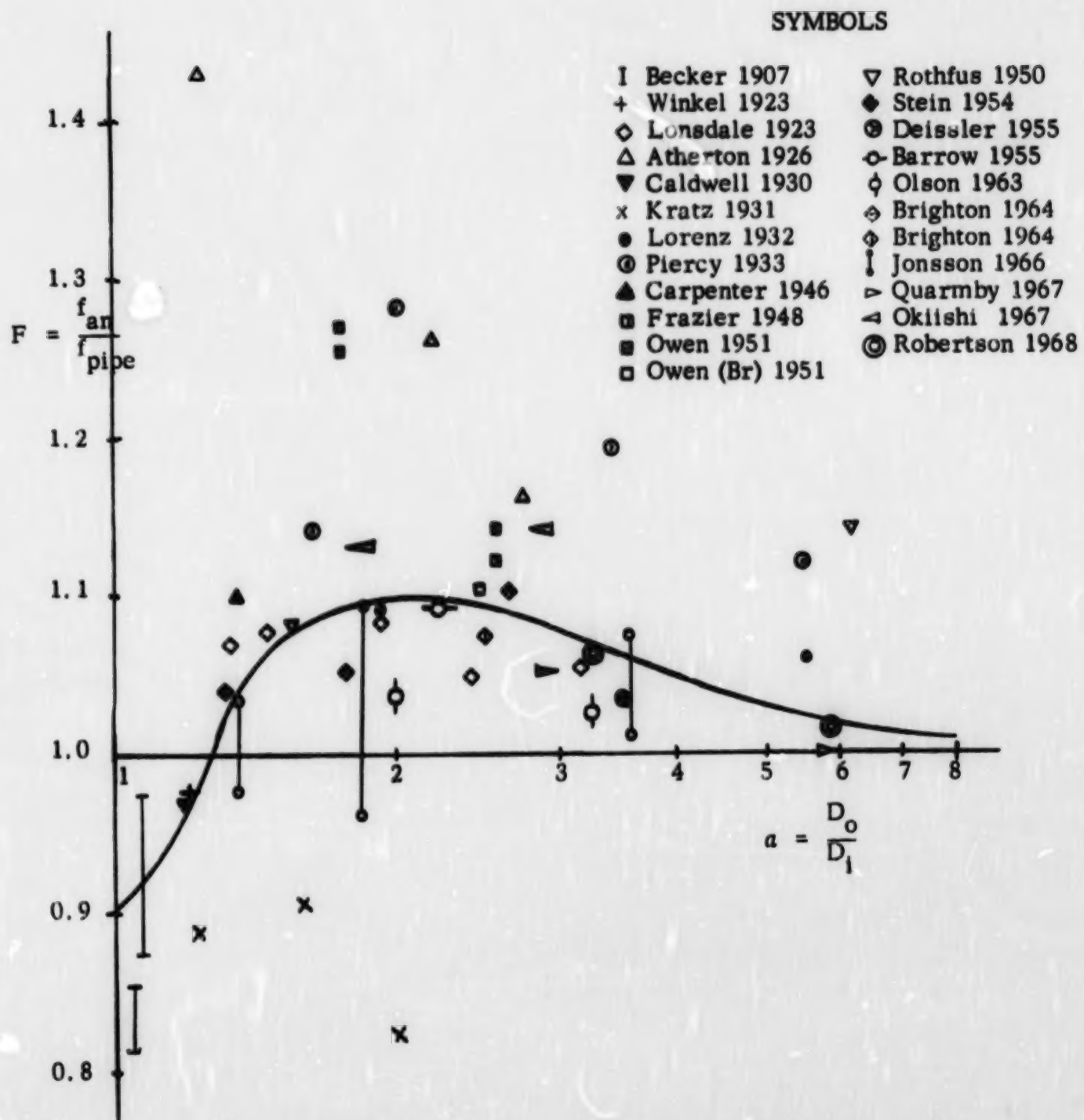


Fig. 1 Turbulent-Flow Friction Factor for Concentric Annulus as a Function of Diameter Ratio.

experimental data available --excepting, of course, the Atherton and Kratz data.

The suggestions of  $F$  dropping close to unity at the higher values of  $a$ , despite values ranging from 1.0 to 1.14 at  $a = 5.5$  to 6.1, is based on the indication of Deissler and Taylor's calculation, the results from the present study, and the intuitive expectation that the effect of the inserted pipe should die off rather rapidly with relative decrease in size of the insert, at least in the turbulent flow regime. It should be noted that the proposed analyses of Phase II of this general study following Deissler and Taylor should settle this point unequivocally. The exact levels of  $f$  may not be absolutely certain from such an analysis, but the trend with  $a$  should involve no uncertainty.

**Eccentricity Effect.** The variation of annular-conduit friction factor with eccentricity is a function of  $\epsilon$  and  $a$ , as has been clearly demonstrated for the laminar-flow case. Early in this study it was surmised that the eccentricity effect could be correlated via the effect of diameter ratio  $a$  on it at maximum eccentricity ( $\epsilon = 1$ ) and the effect of variation with eccentricity ( $0 < \epsilon < 1$ ) independent of  $a$ , thus

$$(10'') \quad E = \frac{f}{F f_{\text{pipe}}} = 1 - \Delta(1 - E_1)$$

where  $E_1 = \frac{f(\epsilon = 1)}{f(\epsilon = 0)} = \text{func}(a)$  represents the maximum eccentricity effect and  $\Delta = \delta f / \delta f_1$  is the ratio of the change  $\delta f$  in  $f$  from the concentric value at a given  $a$  to  $\delta f_1$  that for  $\epsilon = 1$ . It was postulated that  $\Delta$  should be independent of diameter ratio; now  $\Delta$  varies from 0 to 1 as  $\epsilon$  varies from 0 to 1 so that for  $\epsilon = 1$ ,  $E = E_1$ . As will be noted the results of Jonsson and Sparrow (1966) contravene this assumption. However, there still appears to be some advantage in attempting to separate the two effects in the above fashion.

The variation with diameter ratio  $a$  of the friction factor for maximum eccentricity --as exemplified by  $E_1$  --is presented in Fig. 2 for the laminar flow analytic results and the turbulent-flow experimental results. The laminar flow predictions are presented since it is felt that they should, at least, suggest the mode of variation to be expected in the turbulent case. This they do but some divergence appears between the predictions of Piercy, Hooper, and Winney (1933) and those of Caldwell (1930) and of Redberger and Charles (1962). The last named predictions contain the most extensive range in variables but the interpretation given for them is uncertain due to the small-scale presentation of their final results. Since the Caldwell prediction agrees with these, the Piercy predictions would seem to have been miscalculated somehow.

$$\epsilon = \frac{2 \Delta c}{D_0 - D_1} = 1.00$$

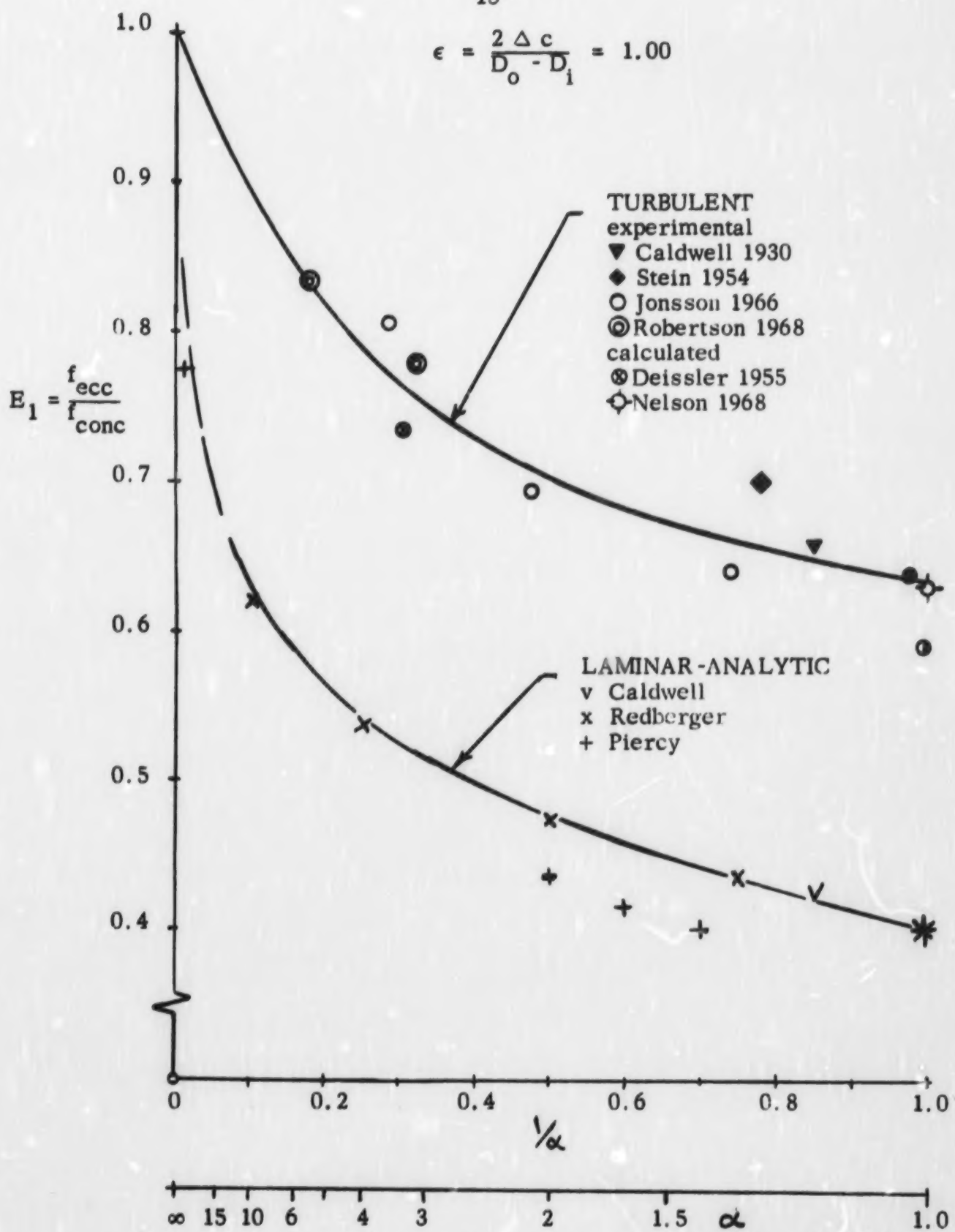


Fig. 2. Effect of Diameter Ratio on Maximum Eccentricity Factor.

In the case of the turbulence results a definite trend independent of  $a$  is suggested by the data of various experimentors, together with values calculated by Deissler and Taylor (1955) for  $a = 3.5$  and by A. R. Nelson for  $a = 1.01$ . This latter calculation is based on the wide-rectangular channel turbulent friction result given by Eq. (11) and is detailed in Appendix B. It serves to anchor one end of the suggested trend line, and appears to be in rather good agreement with the experimental indications for low  $a$ .

The variation in friction loss with eccentricity, i.e., for  $\epsilon \neq 1$ , is found not to be well correlated by the supposition that  $\Delta$  of Eq. 10' should be independent of  $a$ . In fact, as apparent from Fig. 3, a consistent variation in  $\Delta$  with  $a$  appears in the turbulent-flow results of Jonsson and Sparrow (1966) for  $a = 1.33, 1.78$  and  $3.56$ . Other less-extensive data from Caldwell (1930) at  $a = 1.18$  and from the present study seem in reasonable agreement as do the results of Tao and Donovan's (1955) study\* at  $a$  close to unity. Cross plots of the trend lines for three  $\epsilon$  values are indicated in the insert. A moderate, but consistent, effect of  $a$  is apparent but the main parameter controlling  $\Delta$  is still obviously  $\epsilon$ , at least for  $a$  in the range of 1 to 5 ( $1/a > 0.2$ ).

For laminar viscous-flow occurrences indication of the variation of  $\Delta$  with  $\epsilon$  and  $a$  should be available analytically. The fine-clearance ( $a = 1$ ) result is given by Eq. 9 and is indicated in Fig. 3. The trend of variation is not greatly different from the corresponding turbulence result of Tao and Donovan. Some suggestion as to the effect of  $a$  on the  $\Delta$  vs.  $\epsilon$  variation should be available from laminar theory. Unfortunately the most extensive study of laminar eccentric annular flow (Piercy, Hooper and Winney) seems to have involved some errors of calculation, so only Caldwell's (1930) prediction for  $a = 1.183$  is indicated. Although this latter result deviates appreciably from the  $a = 1$  curve, it is not easy to infer the occurrences at larger  $a$  values. The Piercy, Hooper and Winney analytical results suggest no great deviation from the  $a = 1$  predictions.

The laminar-viscous solution is seen to offer no adequate suggestions as to the effect of  $a$  on the  $\Delta$  vs.  $\epsilon$  correlation. The turbulent flow analysis of Deissler and Taylor (1955) should be useful here. However they only considered the  $a = 3.5$  case and, as apparent in Fig. 3, their results indicate little deviation from the Tao and Donovan (1955) results for  $a = 1.00$  but appreciable deviation from the Jonsson and Sparrow (1966) results. Further study of the effect of  $a$  seems necessary before the nature of the  $\Delta$  vs  $\epsilon$  occurrences can be properly established for turbulent flow. For the purposes of interpreting the results of the present study, a tentative turbulent line--

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\* The line depicting these results has been evaluated from their very crude plot and this is not very precise.

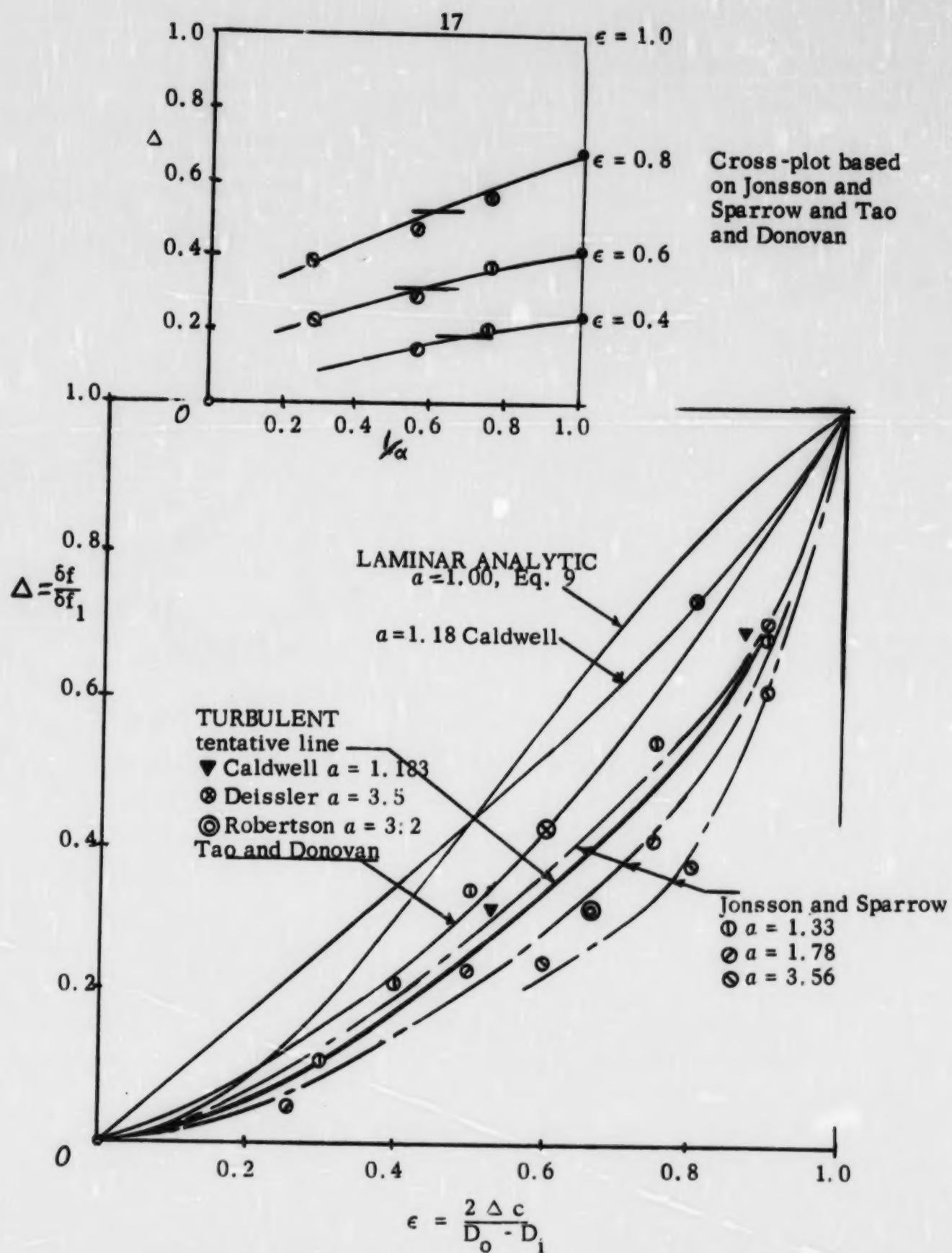


Fig. 3. Eccentricity Effect on Change in Friction Factor

shown dashed in Fig. 3--has been taken as representing the averaged results independent of  $a$  variation. The indication of this tentative line on the inserted  $\Delta$  vs.  $1/a$  cross plots is given by the horizontal dashes. It is seen to represent these results reasonably well for  $1/a > 0.2$ .

## II. EXPERIMENTS ON TURBULENT FRICTION

This report not only reviews the available information on annular friction but also presents some additional experimental data. That data has already appeared in Figs. 1, 2, and 3; although only a few points are involved, they are seen to be important in defining the trends of the various effects. It is important, therefore, to indicate clearly the mode of obtaining this new data so as to establish its validity.

### Experimental Setup and Pipeline

The friction data on turbulent flow in annuli reported herein were obtained in a 100 foot long line of 6 in. diameter pipe located in the lower level of the Hydraulics and Fluid Mechanics Laboratory. This is the same pipe as used by Poyser (1940), Frazier (1948) and Owen (1951) and advantage was taken of the various facilities which these researchers had added.

The general arrangement of the pipeline, water supply and flowrate measuring system is indicated in Fig. 4. The flow to the pipeline was supplied by one or more of the laboratory supply pumps (1000 and 2000 gpm) taking water from the sump under the floor and pumping into the 6 ft diameter standpipe, which is provided with a skimming weir near the top so as to hold the head on the pipeline to 54 ft 9 in. Overflow indicators on the return line from the skimming weir served to indicate when enough pumping capacity was being employed to produce the needed flow. The test line, which had a length of some 110 feet, took off from the side of the standpipe at an elevation of over eight feet from the floor and discharged into the measuring flume through a control valve at its end. The details of this line are discussed separately below. The water then returned to the sump via the flume and was metered via its head on flowing over the weir at the end of the flume. The details of the flume, weir and calibration are noted separately below.

### Test Conduit

As has been noted the test conduit involved the same 6 in. pipeline as employed by Poyser, Frazier and Owen but with some modifications. These involved changes in the pipe sections immediately preceding and following the reaches over which the friction loss was measured, and changes in the supports used for inserts and in their sizes. The various features are detailed in the following subsections.

The 6 in. Pipeline. The general nature of the pipeline is indicated in Fig. 4. It started as a 10 in. line welded into the side of the headtank and a 45 degree miter bend in the horizontal plane immediately followed. This was followed by a 10 in. gate valve, for cutoff purposes (full open during all tests), a short section of 10 in. pipe with a drain

fitting and then a contraction to 6 in. The studies of Poyser with this setup indicated considerable unsymmetries --presumably resulting from the miter bend, which should have been provided with turning vanes --so that the enlargement to 9 in. pipe with honeycomb nest of 3/4 in. tubes was added. This then yielded good symmetry in velocity profiles.

As studied by Frazier, the pipeline, following the honeycomb and contraction back to 6 in., consisted of five lengths of 6 in. steel pipeline each of approximately 20 foot length. Each of the joints between these lengths contained a piezometer ring, of one inch length in the flow direction with four holes evenly spaced around the periphery. This was carefully mated and aligned to the ends of the associated pipe lengths so that minimal joint disturbance occurred. A pair of dowel pins served to maintain this alignment on disassembly and reassembly of the joints. The bolted joints were sealed with gaskets; care was taken to assure non-projection of the inner edges of the gaskets into the flow region.

In the present experiments only four of these lengths of the steel pipe were employed, in order that the last piezometer ring could be kept away from the flow control valve at the end of the line. As shown in Fig. 4, the test lengths of pipe were preceded by a 4 ft 1 in. and a 9 ft 9 in. length of 6 in. pipe, these particular lengths being some which were available. The four special 20 ft pipe lengths were then followed by a 5 ft 2 in. length of 6 in. steel pipe and the gate valve used for flow control. A short length of pipe then directed the flow into a 6 ft wide chamber through which it then fell to the surface of the weir flume shown in Fig. 4. The five ft length of pipe between the valve and end of the last 20 ft length was deemed sufficient to eliminate any possibility of the valve, at partial opening, disturbing the pressures at the end of the 20 ft. section and thus a piezometer ring could be employed there.

Before the present experiments were carried out, the 6 in. pipe line and the honeycomb were thoroughly cleaned to remove the rust, corrosion and deposits which had collected over the years. The 20 ft test sections of pipe were cleaned with a pipe cleaning brush ("bristles" of spring steel about 1/8 in. in width) rotated by an electric motor as it was dragged back and forth in the pipe. This appeared to clean the pipe out thoroughly with no residual indication of where the deposits had collected on the bottom. As will appear from the hydraulic tests, this did not mean the pipe was smooth by any means. The interior surface had the appearance of a fine sand finish.

Precise knowledge of the inside diameter of the pipe is vital to correct evaluation of the friction measurements. Some micrometer measurements, taken to obtain an idea of the inside diameter and with no hope of exact definition, indicated values of the

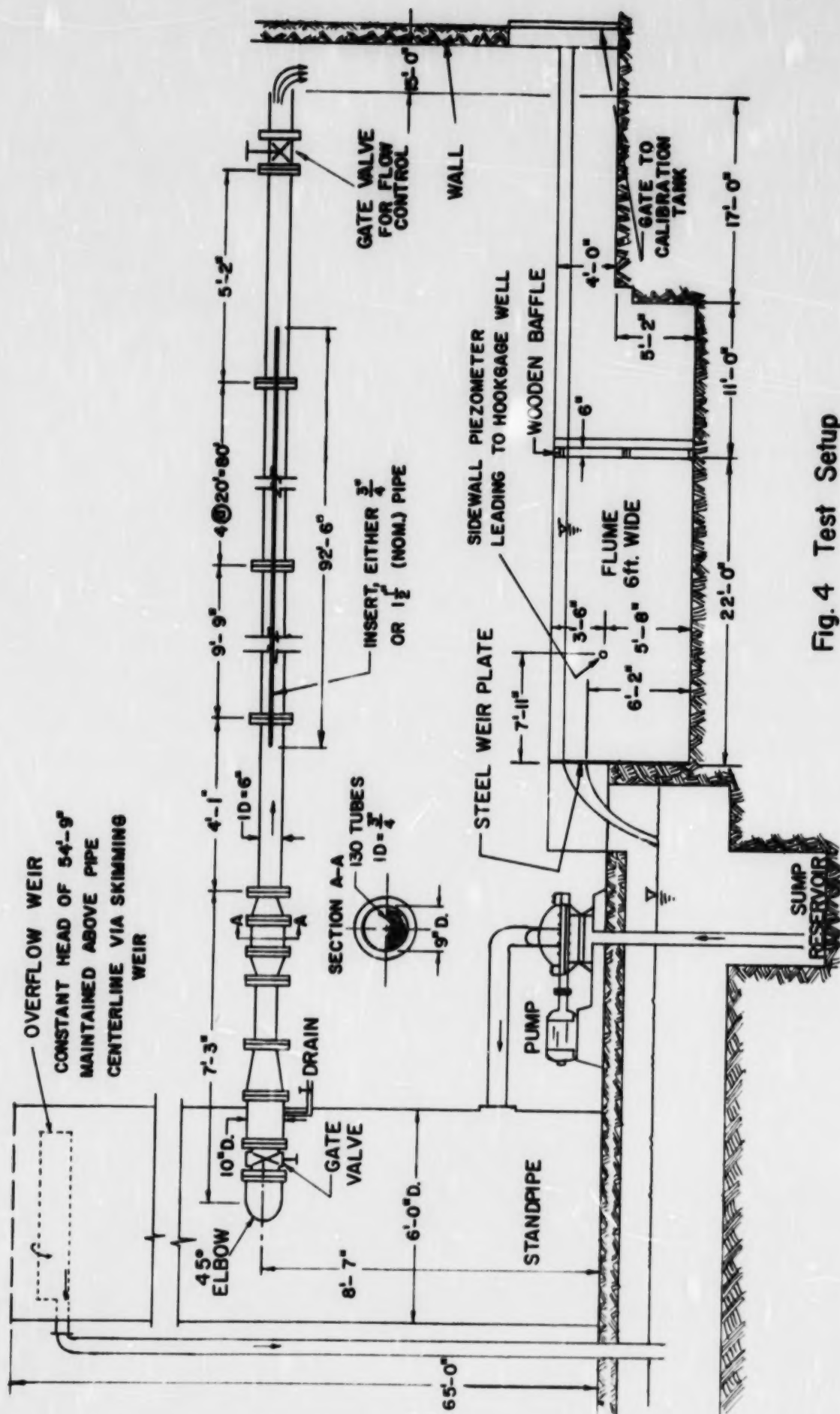


Fig. 4 Test Setup

order of 6.054 to 6.146 inches with an average (for pipe sections 12 and 23) of  $6.92 \pm 0.020$  in. The effective diameters of the last three sections of pipe (sections 23, 35, 45) were obtained by capping one end, standing the pipe section on this cap on a scale and then weighing the pipe empty and full of water. The weight of water divided by the exact length of pipe involved then led to an indication of the sectional area and diameter, averaged for this length. The weights of water involved were 250 lbs and after the measurements the scale checked, at this weight, to within 1/2 lb. The areas and diameters thus found are

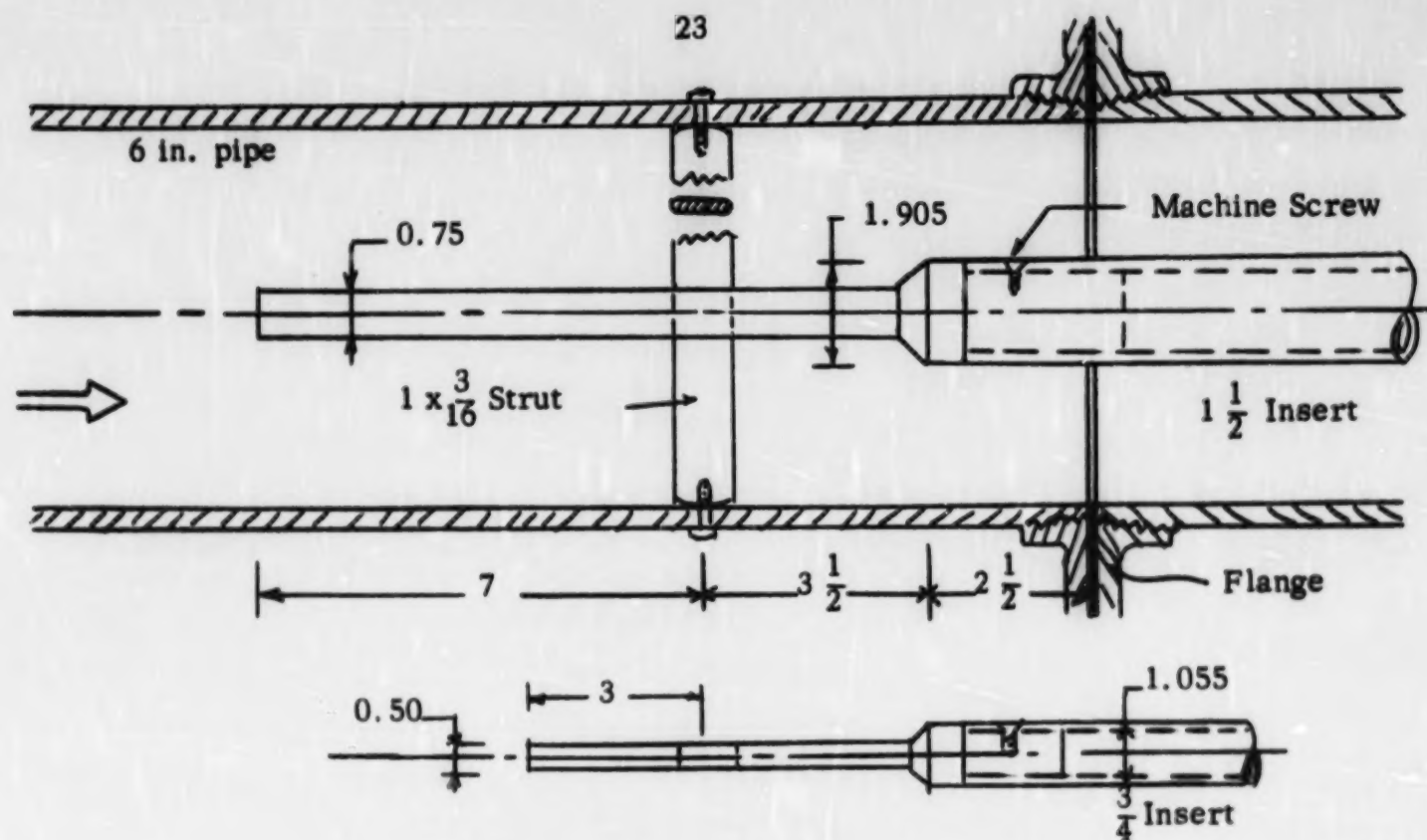
|         |     |        |                         |            |          |
|---------|-----|--------|-------------------------|------------|----------|
| section | 23, | area = | $0.2019 \text{ ft}^2$ , | diameter = | 6.08 in. |
|         | 34  |        | 0.2042                  |            | 6.12     |
|         | 45  |        | 0.2013                  |            | 6.07     |

These numbers were appropriately averaged in reducing the test data. The diameters are seen to be in excellent agreement with the micrometer measurements, a surprising fact in view of the roughness of the inside surface.

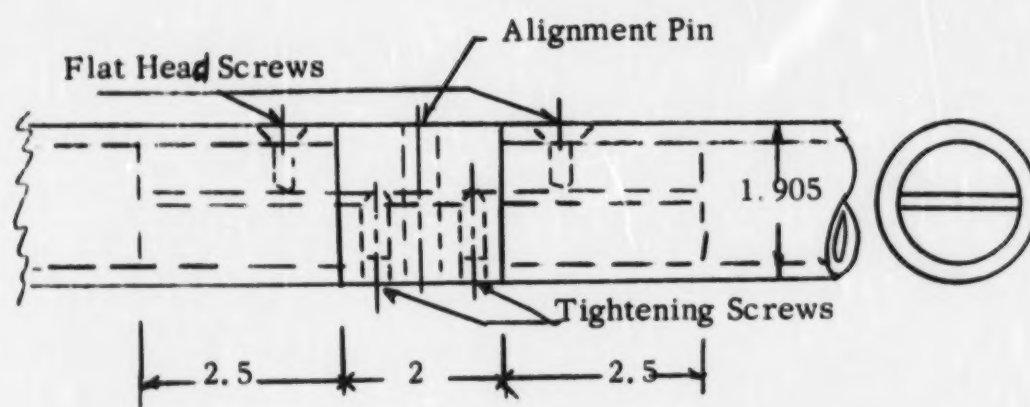
The Inserts. The annular conduits were made by inserting smaller pipes inside the 6 in. line. Two insert sizes were used namely 3/4 in. and 1-1/2 in. galvanized pipe. Careful measurement indicated a variation in diameter from 1.047 to 1.069 in. for the smaller, with an average value of 1.0554 in. for section 25 and 1.0537 in. for section 35. In the case of the larger insert the outside diameters were found to be 1.904 for section 35. Thus the tests involved a values of 5.79 and 3.20.

The length of insert employed was 92 ft 6 in., as indicated in Fig. 1, somewhat less than the available length of 6 in. pipeline. The insert extended either 9 or 12 in. into the 4 ft length of 6 in. pipe following the honeycomb. The start of the insert, in each case, was a blunt rod of smaller diameter, followed at 6 or 10 inches by a 45 degree shoulder to the main insert diameter, as indicated by the sketches of Fig. 5a. The purpose of these blunt ends and shoulders was to initiate rapid boundary-layer growth on the inserts so that fully-developed conduit flow resulted as soon as possible. The inserts were cut to appropriate lengths so that they could be mounted in the 6 in. line sections on assembly of the pipeline and then suitably joined. The junctions employed for the inserts are indicated in Fig. 5b. After assembly the joints were bound with vinyl tape so that no flow could occur inside the inserts. The mounting of the inserts in the 6 in. pipe is discussed separately below.

One important feature of the conduit assembly deserves comment, although no fluid mechanics is involved. This is the matter of the handling the assembly of the line.



(a) Insert Noses: Details and Front Strut



(b) Plug Connection for  $1 \frac{3}{4}$  Insert

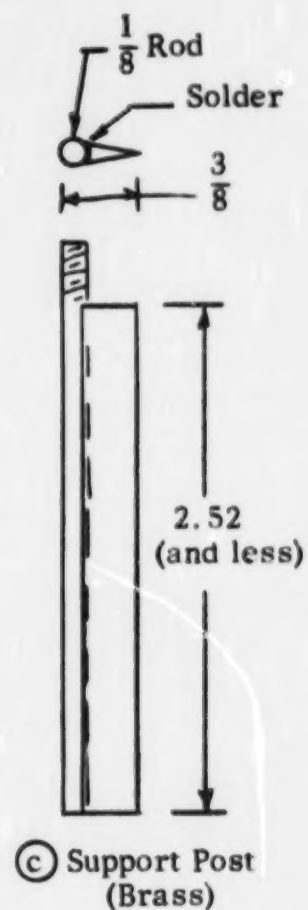


Fig. 5. Insert Connection and Support Details

To insert the inner pipes in the 6 in. one and to change their eccentricity, it was necessary to almost completely disassemble the line. To achieve this, an overhead track with small trolleys supporting the 6 in. pipe sections via chainhoists was mounted over the pipeline. It was found necessary to disassemble the 6 in. and insert pipes at each joint in changing the eccentricity; on the first time this change was made, a rush job was tried without complete disconnecting of the insert joints and many of the mounting supports were badly damaged.

The end of the pipeline near the control valve was tied to a rigidly mounted timber with a U strap. In some of the first tests, before this was done, appreciable oscillation in the manometers appeared at intermediate flow rates; this was greatly reduced on tying down the end of the pipe.

Mounting of Inserts in Pipe. The importance of supporting the insert properly so that, at all points, it is exactly concentric or has a prescribed eccentricity has been noted. This is particularly important in the laminar case, but the potential effect of some slight unplanned for eccentricity on the apparent friction factor in turbulent flow is not insignificant. This factor is not simply taken care of by using many stiff supports, since too many of these, even if well streamlined, will add significantly to the total friction drop in the conduit. The delicate task of getting adequate support without appreciably adding to the friction has been sidestepped by the use of vertical conduits by some researchers. Then the insert has been held in place via tension applied at its ends. In view of the lengths involved for conduit of any size, one is apt to cut short the approach length. Furthermore the resistance of the insert to small lateral forces is not likely to be large, so that it would seem that unknown misalignments are still possible. In any case the present test line was horizontal so that the problem of adequate support with minimum drag had to be met.

In order to decide how close the insert supports should be placed, calculations were made of the deflection of the two inserts. The ninety foot long insert was considered to be a continuous beam on simple supports. With support at every joint, 20 ft lengths are involved; under its own weight the smaller insert will then have a maximum deflection of 0.82 in. and the large one of 0.23 in. These deflections are, of course, excessive. Doubling and tripling the number of supports reduces these deflections to 0.05 and 0.01 and 0.014 and 0.003 in. respectively. In view of the uncertainty and variability in the diameter of the 6 in. pipe it was decided that 0.01 in. was the best that one could hope for in setting the supports. Therefore the support spacing was set at about 6 ft 8 in. This required two interior supports in each of the 20 ft pipe lengths.

It was not deemed feasible to surther section the 6 in. pipe so that the interior supports had to of such nature that the inserts could bc slipped in with them attached. Two types of interior supports were used. These were one legged (vertical) and two legged (at 45 degrees to the vertical) posts threaded into the inserts and cut to the proper length to give the specified clearance. Use of these does assume that the weight of the insert exceeds any hydrodynamic force which may try to distrub its orientation. These posts were made of 1/8 in. brass rods streamlined to a length of about 3/8 in. as sketched in Fig. 5c. At each joint in the 6 in. line the inserts were tied in their desired location by a Y shaped yoke of three steel wires of 0.032 in. diameter. These wires passed through 0.040 in. holes in the walls of the 6 in. pipe where they were caught and tensioned by machine screws which permitted one to hold the insert in its desired locale. The settings of these screws were fixed with locknuts. Leakage through the holes in the pipe wall was sealed with Duco cement, although on occasion the seal broke and small amounts of water dripped out.

The additional friction induced by the supports has been estimated from informa - tion on the drag coefficients of these shapes --this analysis indicated a distinct advantage of streamlining the posts, since without the streamlining the additional friction is about twice as great. On the basis of three posts and one three-wire yoke support, the additional friction added to the flow has been estimated at 3 percent of the measured  $f$  value of 0.018. Since the supports were about as fine and streamlined as possible and possible measurement errors certainly must be the order of a percent or so, this frictional increase due to the insert supports was deemed negligible.

#### Flow Rate Measurement

It was not possible to positively meter the rate of water flow through the test conduit by primary methods and a 90-degree V-notch weir had to be employed instead. This weir was first used by Frazier who stated that the discharge is given by the relation  $Q = 2.54 H^{2.5}$ , where  $H$  is the net head on the crest; he offered no justification for this relation. For the present studies the weir was calibrated in place by volumetri - cally measuring the flow before it passed into the flume.

The Weir and Flume. The general layout of the measuring flume and weir at the end of it is indicated in Fig. 4. The main part of the flume is some 9 ft deep and 6 ft wide. The 3/4 in. steel plate forming the weir plate is mounted flush to another steel plate on the upstream face of the 8 in. concrete wall at the end of the flume. The notch of the weir was some 6 ft 2 in. above the floor of the flume and for the maximum flow, the head

was about 1.3 ft. Some 7 ft 11 in. upstream of the weir plate, a piezometer hole through the sidewall of the flume led the water to a stilling well in which a hookgauge on a traverser, readable to 0.001 ft via a vernier, permitted obtaining the head on the weir. The "zero" of this reading was found both by the reflection and transfer (via a rod with level and auxiliary hookgauge) methods.

A wooden baffle, composed of 36 wooden slats of 1 by 6 in. lumber mounted vertically, with the flow between them in the 6 in. direction, was mounted in slots in the side walls of the flume at a distance 22 ft upstream of the weir. Unfortunately the manner of the pipe discharge into the water surface and the long length of channel involved resulted in a surging motion in the flume which then disturbed the head readings at the largest flow rate. A higher-resistance baffle should also have been employed in the shallower section of the flume, but the exigencies of the test schedule did not permit its installation. Some 11 ft upstream of the wooden baffle, the flume shallowed by 5 ft 2 in. and thence extended 16 ft further to its nominal end in a wall. This wall contained a 2 by 3 ft sliding gate leading into the 199.5 ft<sup>2</sup> surface-area calibration tank.

Calibration. The flow from the pipeline could not be measured by primary means after exit, nor could the flow over the weir be so checked; both for the same reason, that no space was available downstream to set up a volumetric tank or tank and scale system. However it was possible to calibrate the weir via a primary check of the flow rate before the flow entered the flume proper. This was done by running water at some rate through another pipe line into the calibration tank upstream of the weir flume. This flow rate was easily measured directly via the rise of water in the calibration tank when the sliding gate was closed. Once this rate was established, the sliding gate was opened and the flow through the flume and over the weir allowed to establish itself. The inflow rate not having been changed, a definite calibration point--weir head vs volumetrically determined flow rate--was established. In this manner six calibration points for the weir were established; these are indicated in Fig. 6 in the form of the coefficient  $C = Q/H^{2.5}$  as a function of the head  $H$ .

A slowly decreasing coefficient, with increase in  $H$ , is indicated as having been obtained. This does seem to agree well with coefficients found by other researchers on 90 degree weir calibrations, although it diverges some one to two percent from the value given by Frazier. In view of the general agreement with others and the small scatter in the data, the weir calibration indicated in Fig. 6 is deemed quite reliable.

The flow rates involved in the conduit friction evaluations varied between 0.31 and 4.85 cfs. The corresponding weir heads were 0.432 and 1.310 ft; reference to Fig. 6

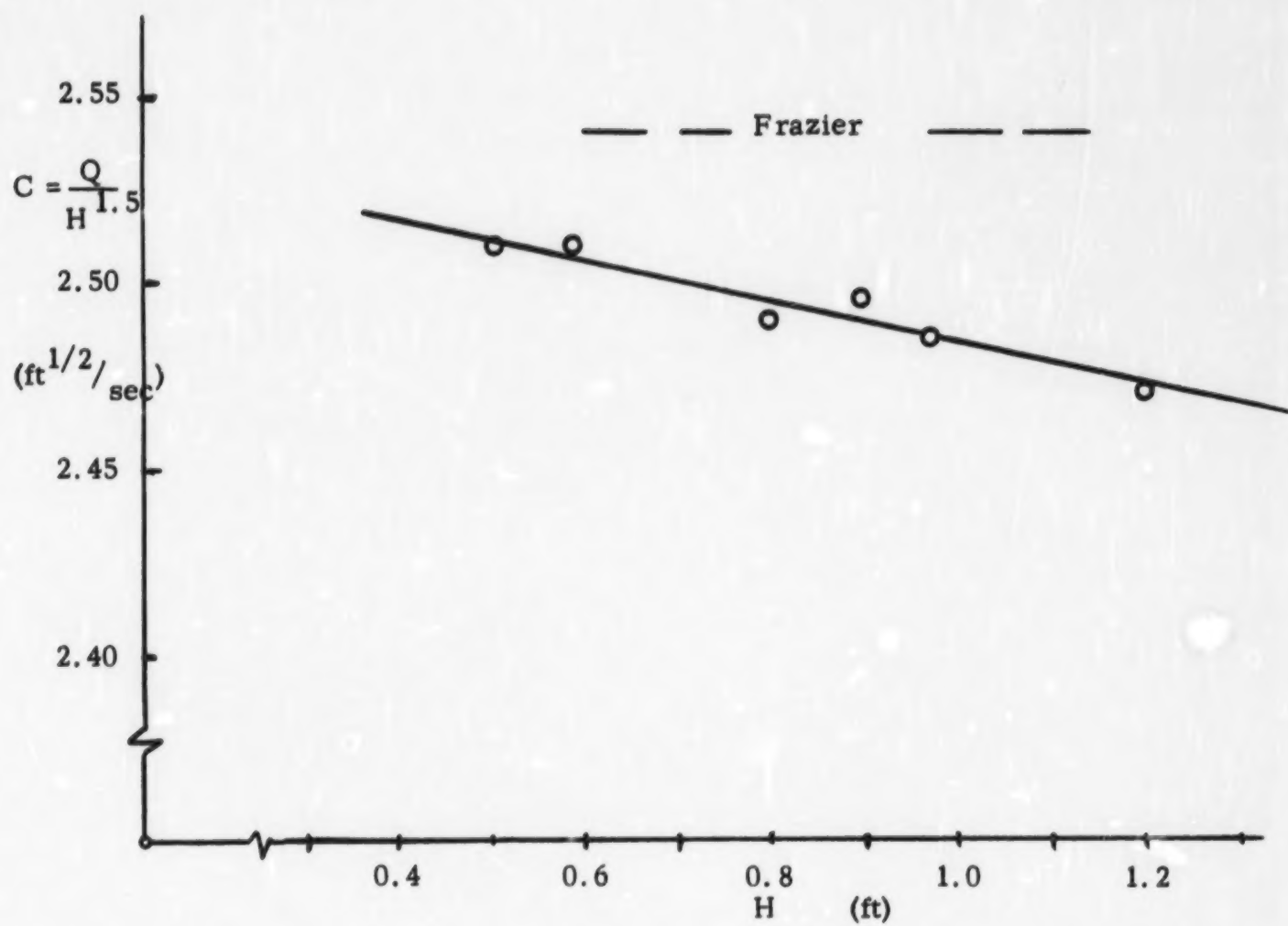


Fig. 6. Weir Calibration

indicates that little extrapolation of the calibration of the weir was involved in covering this range of flows. As has been noted, some difficulty was encountered in reading the weir head at the highest flow rates. Based on the observed maximum and minimum gage readings at the highest flow rate, the rate varied about  $\pm 7$  percent. For this particular friction measurement in the bare pipe (see highest point in Fig. 7) a misreading also appears to have been made. In general the surging was small and is deemed to have had no significant effect.

#### Pressure Drop Measurements

The pressure drops indicating the friction along the conduit were measured with two U-tube differential manometers. For the larger flow rates, and thus pressure drops, a 36 in. mercury-water manometer was used, while for the smaller ones a 4 ft carbon tetrachloride-water manometer was employed. The specific gravities of the gage fluids were taken as 13.60 and 1.59, respectively.\* The manometers were provided with bleed valves and lines, with shut off valves, and were connected to the manometer lines from the test pipe via hoses and quick-disconnect fittings. This permitted switching either manometer to any set of piezometer lines. The manometers were, of course, bled after every such change.

The use of piezometer rings positioned between the 20 ft lengths of 6 in. pipe and at the end of the last one has been noted. At four equidistant points around their inside periphery, these contained brass plugs through which the 1/16 in. piezometer holes had been drilled. The four holes were joined via 1/4 in. pipe fittings and rubber hose to form a loop about the piezometer ring section. At the approximate apex of this loop a bleed valve was provided; this was used frequently throughout the tests. In the lower quadrant a tee fitting provided connection to one or more lengths of 1/4 in. pipe which were run along the pipeline to a point about midway along the measurement reach. Here each of the four piezometer lines ended in a quick-disconnect fitting for attachment to the manometers via hoses as desired.

There were four piezometer rings, one at the end of each of the 20 ft lengths of 6 in. pipe. There were numbered 2, 3, 4, and 5 in the flow direction and the pipe lengths termed 12, 23, 34, and 45. In all of the tests the pressure drops 2-5 and 3-5, i. e., over lengths 23, 34, 45 aggregating 59.22 ft and 34 and 45 aggregating 39.27 ft, were measured. In some of the runs the drop over individual lengths were checked. Some divergencies appeared between these various results and it was finally deemed

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\* Actually the specific gravity of  $\text{CCl}_4$  decreases with increase in temperature, being 1.595 at 66 F and 1.585 at 75F, and the water temperature during the tests varied from 68 to 75F.

most appropriate to use the 3-5 length of conduit, thus providing an approach length of some 54 ft for the 6 in. pipe and 50 ft for the annular conduits. These correspond to 106 and 119 diameters, which should be adequate. The pressure drop measurement length of 39.27 ft was still sufficient to yield manometers readings of at least 0.20 ft at the smallest flow rate studied.

Very little trouble was encountered with manometer pulsation, except in the situation noted above in which the pipeline was not well tied down. The manometer zero readings were carefully checked before and after each series of runs.

#### Test Schedule

The measurements of annular pipe friction were conducted in two parts as dictated by the effort involved and other commitments of the investigator. The specific schedule followed is given in the Table II below.

TABLE II: Schedule of Tests

|                          |                 |
|--------------------------|-----------------|
| Weir Calibration         | 5-6 June 1967   |
| Friction of 6 in. Pipe   | 8 June 1967     |
| Friction of 3/4 Insert   |                 |
| concentric               | 16 June 1967    |
| full eccentric           | 17 June 1967    |
| Friction of 1-1/2 Insert |                 |
| concentric               | 6 October 1967  |
| 2/3 eccentric            | 20 October 1967 |
| full eccentric           | 1 November 1967 |
| Friction of 3/4 Insert   |                 |
| with simulated hangers   | 6 January 1968  |

The time between the 6 in. pipe test and the first insert tests was spent in assembling the small insert in the pipe line. The change between the concentric and full eccentric position of this insert was accomplished overnight due to the press of other commitments which, however, delayed testing of the larger insert until fall.

The pressure drop for each conduit arrangement was measured for as large a range in flows as permitted by the test setup. The maximum flow rate was fixed by the overall friction loss and the available head; maximum velocities of 20.8 to 23.9 fps were achieved. The lowest velocities, about one tenth the maxima, were fixed by the smallest manometer readings deemed reliable. In several of the test series, the pressure drop

at one given flow rate was measured with both manometers; generally these indications agreed closely, when converted to the same base, i. e., feet of water.

### III. EXPERIMENTAL RESULTS ON FRICTION

The flow rate in the conduit, and thus the flow velocity, was found from the weir head readings via the relation  $Q = CH^{2.5}$  with the coefficients  $C$  obtained from Fig. 6. Thus the flow Reynolds number, as defined by Eq. 6, was obtained in terms of the kinematic viscosity specified by the water temperature measured during the tests. The friction factor was evaluated from the pressure drop readings and flow velocities via Eq. 5. The basic data is presented in tabular form in Appendix C. The correlations of the friction data, in its variation with Reynolds number and geometric parameters is presented below.

#### Friction in 6 in. Pipe

The variation with Reynolds number of the friction factor of the 6 in. pipe is needed to serve as the reference for the annular conduits, to indicate the roughness of the inside of this pipe and to help establish the general reliability of the results offered. Conventional representation, as a Blasius-Stanton chart with reference lines for smooth pipe and transitional--smooth to fully rough--lines following Colebrook's function, of the 6 in. pipe frictional data is presented in Fig. 7. The pipe is seen to be definitely rough, as expected from inspection of its interior surface, and to have a relative roughness  $k/D$  of about 0.0007. This corresponds to an equivalent Nikurase sand-roughness of 0.00035 ft, a not unreasonable value in view of that for steel of 0.00015 and cast iron of 0.00085 ft.

The results of this pipe test do not agree perfectly with the Colebrook transition function, thus  $k/D$  apparently varies from 0.0006 to 0.0007 with increase in Reynolds number. In view of the usual difficulty of evaluating this factor for data in the transition range before the fully-rough flow condition is achieved, this variation is not excessive. Somewhat more disturbing are the excursions of two data points from the smooth trend of the others. It is not difficult to decide that these measurements must be in error, but it is much harder to ascertain that cause. It is true that the data point at the highest Reynolds number involved excessive surging in the weir flume and that at about  $4 \times 10^5$  involved some surging in the manometers, due to the pipe not being well tied down, but these may only be partial explanations. Another possibility is that the manometer was misread.

The estimate of the effective roughness of the 6 in. steel pipe line as  $k = 0.00035$  ft has been indicated. In the Introduction the desirability of matching the roughness of the pipe and insert was noted. Unfortunately there is no simple way to evaluate the

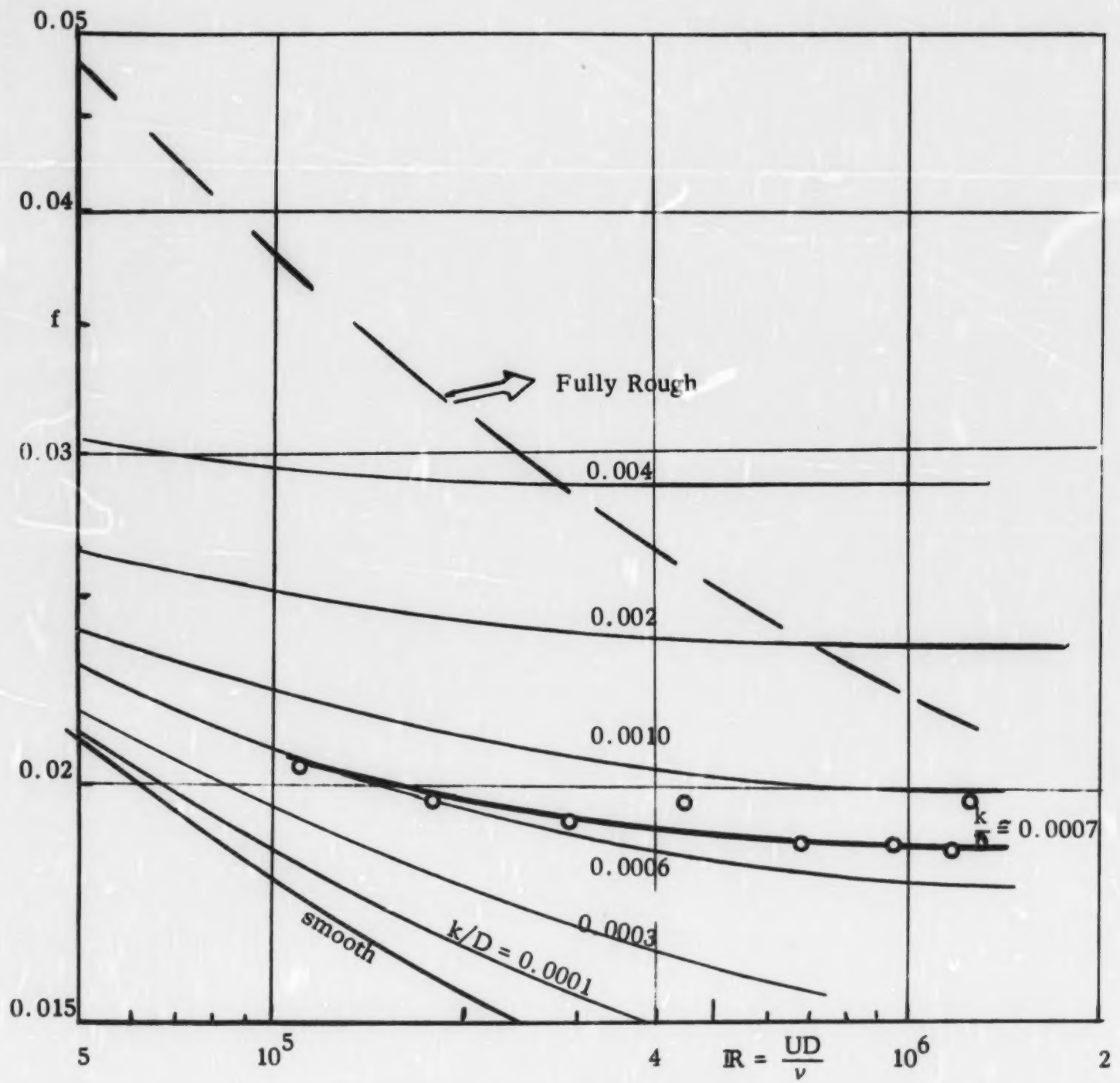


Fig. 7. Blasius-Stanton Chart for Friction in 6 in. Pipe

roughness of the outside surface of the inserts; however, the accepted value of the effective roughness of galvanized iron is 0.0005 ft. Fortunately this is not too far from the roughness found for the pipe, so that one can only hope that little effect of the difference in the two roughnesses was involved.

#### Friction of Conduit with Inserts

For ease of comparison, the friction factor data of the annular conduit with the 3/4 insert (1.05 in. diameter) is presented in Fig. 8 in the form of Blasius-Stanton charts but with the friction factor plotted on a linear scale, rather than logarithmic as in Fig. 7. The same general trend of  $f$  decreasing with increase in  $IR$  appears, except that for the eccentric case there seems to be some drop off below  $3 \times 10^5$ . Whether this is real or merely a vagary of the measurements is not certain. It hardly seems worthwhile to repeat the test to ascertain which it is and, in the general consideration of the annular friction effects, this occurrence is ignored. Also at the highest Reynolds number the friction factor seems to level off at a constant value suggesting an essential achievement of the fully-rough flow conditions.

The corresponding friction factor results for the larger insert are presented in Fig. 9 from which the trend with Reynolds number is seen to be similar to that found with the smaller except for (relatively) slightly higher values of  $f$  at the lowest Reynolds numbers tested.

These two sets of results for friction in annuli indicate a definite decrease in friction with increase in  $a$  and  $\epsilon$ . In order to bring this out more clearly values of  $f$  at  $IR = 2 \times 10^5$  and  $10^6$  have been read off the trend curve for each situation and are collected in Table III. Besides the  $f$  values found for the various situations this table lists values of  $F$ ,  $E_1$  and  $\Delta$  factors as calculated from these results. Only the values of friction at the higher Reynolds number were used in the final correlations in Figs. 1, 2, 3 and 10.

#### Diameter and Eccentricity Correlations

The effects of diameter ratio  $a$  and eccentricity  $\epsilon$  on the turbulent friction in annular pipes has been indicated in Figs. 1, 2, and 3. There the new data obtained from the present experiments, as just detailed has been included. As appeared in the discussions associated with these figures, this new data aided significantly in deciding where the trend lines should go and in verifying  $\epsilon$  trends found, although these are not yet clearly settled.

This present data, together with the general correlations, yields a clear indication of the deviations from the hydraulic-radius concept experienced in annular conduits.

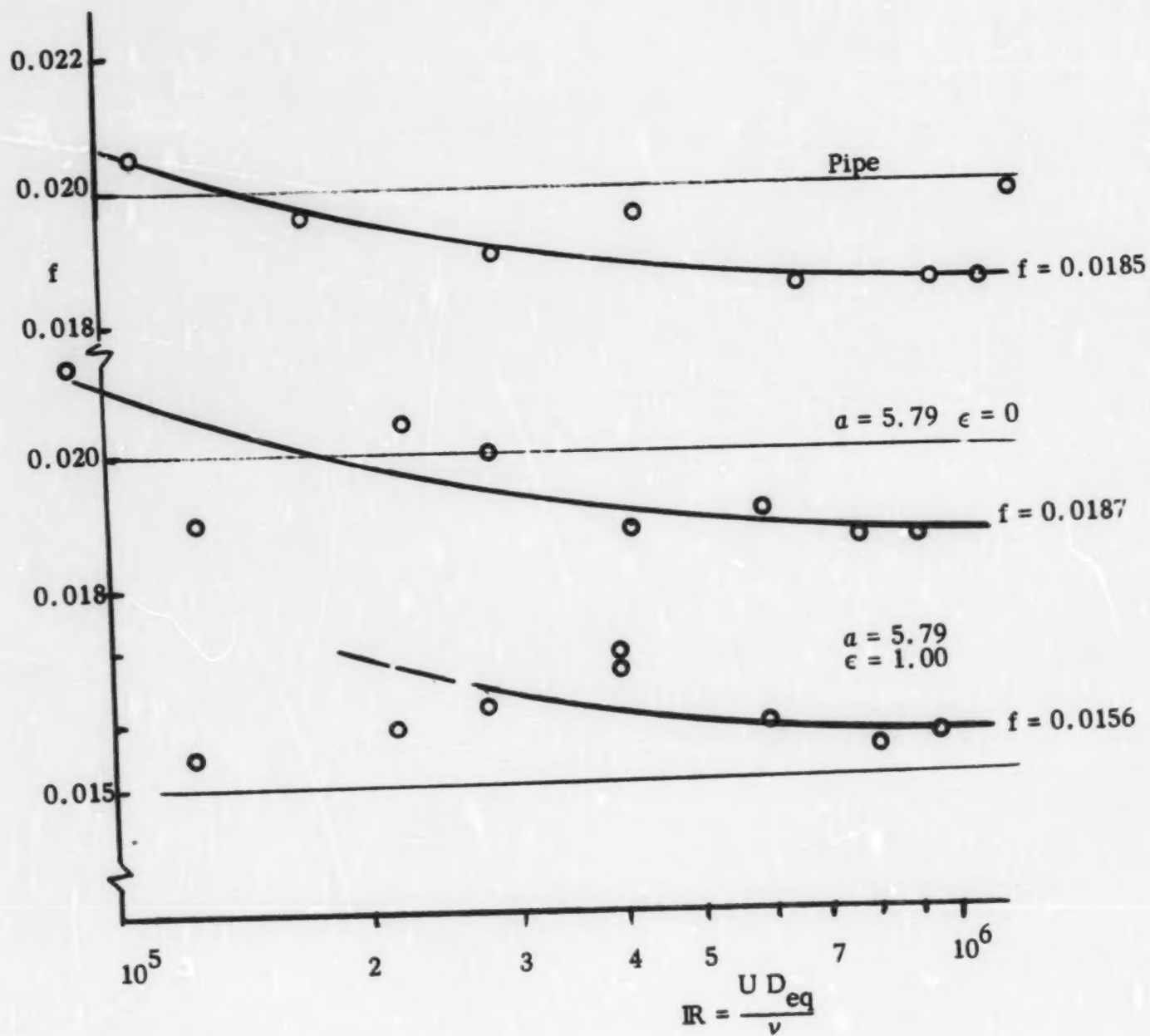


Fig. 8. Turbulent-Flow Friction Data for Pipe and  $a = 5.79$  Annulus in Concentric and Eccentric Arrangement.

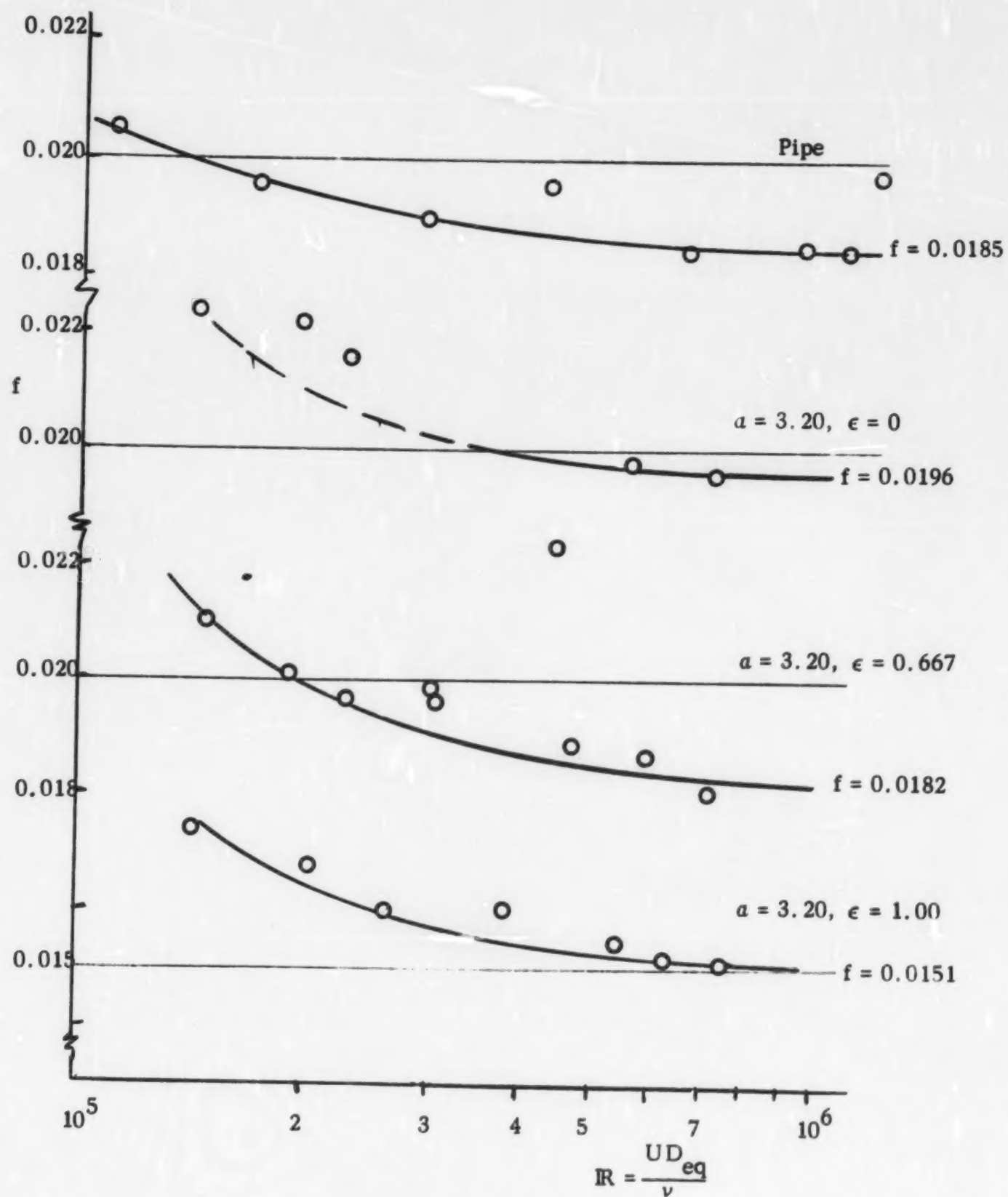


Fig. 9. Turbulent-Flow Friction in Pipe and  $a = 3.2$  Annulus at Various Eccentricities

TABLE III. Summary of Measured Annulus Friction Data

| Annulus  |            | Result at $IR = 2 \times 10^5$ |       |          |          | Result at $IR = 10^6$ |       |       |          |
|----------|------------|--------------------------------|-------|----------|----------|-----------------------|-------|-------|----------|
| $a$      | $\epsilon$ | $f$                            | $F$   | $E_1$    | $\Delta$ | $f$                   | $F$   | $E_1$ | $\Delta$ |
| $\infty$ | -          | 0.0194                         | -     | -        | -        | 0.0185                | -     | -     | -        |
| 3.20     | 0          | 0.021(?)                       | 1.080 | -        | -        | 0.0196                | 1.060 | -     | -        |
|          | 0.667      | 0.0200                         | -     | -        | 0.22(?)  | 0.0182                | -     | -     | 0.32     |
|          | 1.00       | 0.0165                         | -     | 0.785(?) | -        | 0.0151                | -     | 0.775 | -        |
| 5.79     | 0          | 0.0198                         | 1.020 | -        | -        | 0.0187                | 1.012 | -     | -        |
|          | 1.00       | 0.0167(?)                      | -     | 0.843(?) | -        | 0.0156                | -     | 0.835 | -        |

Instead of being independent of conduit shape, the friction factor predictions of Figs. 1, 2 and 3 predict a considerable variation in  $f$ , as indicated by Fig. 10. This depicts the variation in  $FE = f/f_{\text{pipe}}$  with  $a$  and  $\epsilon$  over the  $a$  range of 1 to 8 and for  $\epsilon = 0, 0.667, 0.90$  and  $1.00$ . The  $E$  values for  $\epsilon = 1.00$  are from Fig. 2 and the  $\Delta$ 's for  $0.667$  and  $0.90$  are taken as  $0.40$  and  $0.70$ , according to the tentative line of Fig. 3. The particular experimental data points from this study are indicated as verifying the trends predicted to within one percent.

It is seen from inspection of Fig. 10, that the friction factor of annuli--particularly when the insert is mounted at eccentricities greater than  $2/3$ --may be appreciably less than those for pipe and this may be significant even at diameter ratios above 5. Thus, in a certain sense, the hydraulic radius concept applied to eccentric annuli (or to any annuli above  $a = 7$ ) is rather conservative.

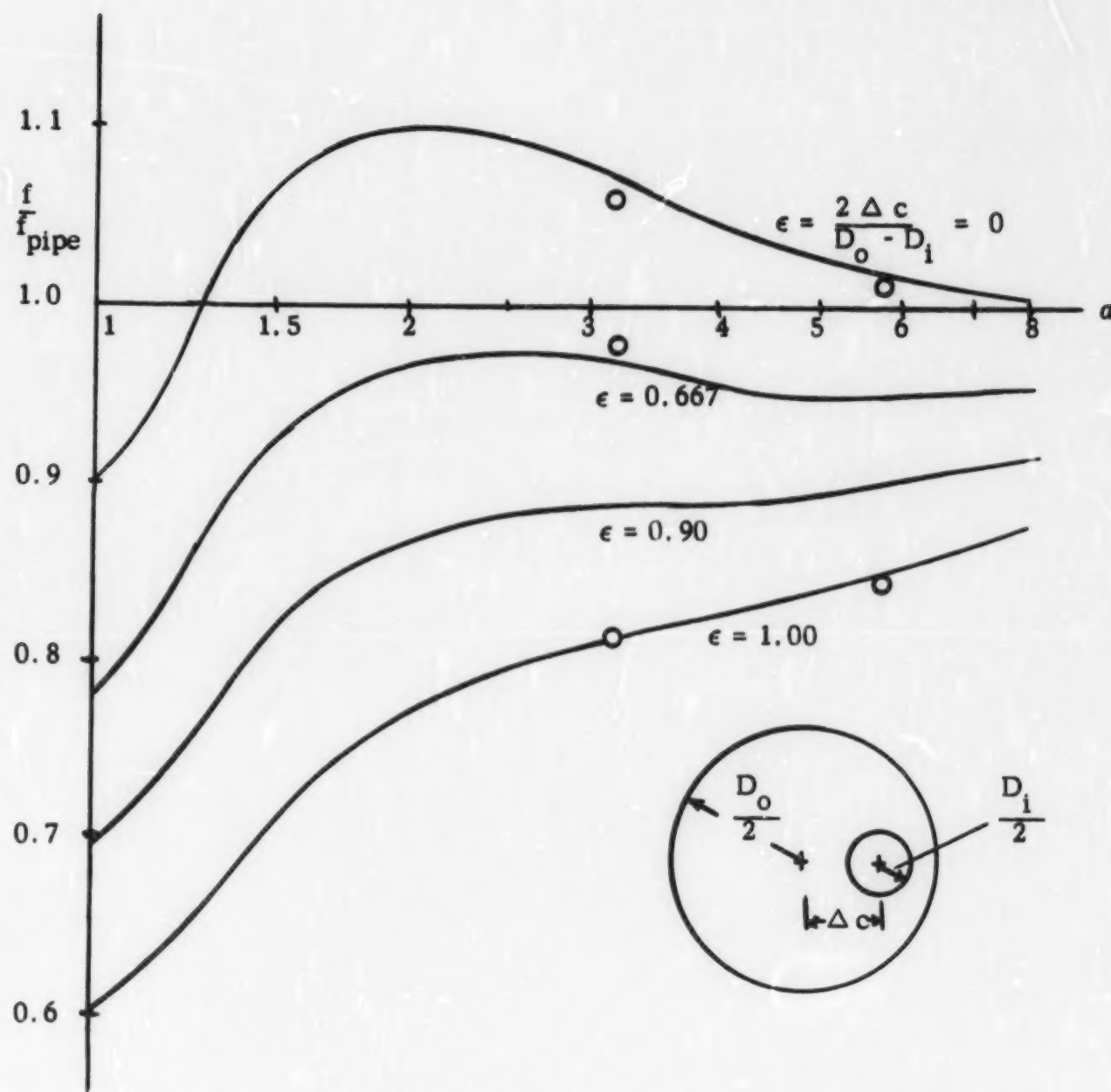


Fig. 10. Deviation of Friction Factor for Annular Conduits from Hydraulic Radius Concept.

#### IV. CONSIDERATION OF SEWER PROBLEM

The effect of an inserted circular cylinder on the friction factor of a pipeline has been considered in Section I and some pertinent experimental data on this matter, which was included in the considerations of Section I, was presented in Section III. In view of the interest in the effect of an inserted pipe on the flow capacity of a sewer, which is the specific application instigating the present study, it is pertinent to reduce the friction factor results to indications of the effect of flow capacity. Some consideration of effects on open-channel flow capacities--i. e., effects on sewers not flowing full--are included as well.

##### Effects of Insert on Flow Capacity

The hydraulic-radius concept, which says that the friction factor defined in terms of the equivalent diameter does not vary with shape of conduit, predicts an 11.8 percent capacity reduction for the smaller insert ( $D_o/D_i = 5.79$ ) of this study. This result is found as follows: From Eq. 5, the flow velocity is

$$U = \sqrt{2g \Delta h} \frac{D_{eq}}{L} \frac{1}{\sqrt{f}}$$

so that the flow is

$$(13) \quad Q = \frac{A\sqrt{D_{eq}}}{\sqrt{f}} \sqrt{\frac{2g \Delta h}{L}}$$

where  $A$  is the flow area. Now, for any given problem under consideration,  $2g \Delta h/L$  is a constant. When  $f$  is independent of the shape of conduit, then  $Q$  is proportional to  $A\sqrt{D_{eq}}$ . For the 6 in. pipe studied this factor is 0.1444 and for the pipe with smaller insert it is 0.1274. These have the ratio 0.882, independent of eccentricity, of course; the reduction is  $1 - 0.882 = 0.118$  or 11.8 percent.

Consideration of the observed effects of eccentricity and diameter ratio on friction factor considerably modifies the above result. Now  $A\sqrt{D_{eq}/f}$  must be considered. This has values of 1.062, 0.928 and 1.016 for the 6 in. pipe, pipe with concentric and with eccentric insert ( $\epsilon = 1.00$ ). Thus the capacity is reduced 12.7 percent by the concentric insert--not greatly different from the value predicted on the hydraulic-radius concept--but reduced only 4.5 percent in the case of full eccentricity. This may seem like getting something for nothing, since the flow area has been decreased by 3 percent and the wetted surface increased by 17 percent. The equivalent diameter is changed 17.5 percent. At full eccentricity, however, the insert is located in a region of

relatively low velocity near the wall rather than under the influence of the mean velocity, as assumed by the hydraulic-radius concept, so that the lesser effect of the insert is not surprising.

The manner of estimating the effect of an annular insert, for diameter ratio or eccentricity other than the case just considered, follows simply from the indications of Figs. 1, 2 and 3 (or 10 for  $\epsilon = 0.667, 0.90$ ) and will not be further considered.

#### Effect of Support Device

In the actual sewer application, the inserted pipe is to be supported near the crown of the sewer. Tentative hanger designs have been developed by Johns-Manville. A sample of one of these, as fabricated to support 3 in. (3.5 in. O.D.) pipe in a 24 in. sewer, was furnished as a model for the simulated hangers used to support the smaller insert in the tests with 6 in. pipe; these two hangers are shown in the photograph of Fig. 11 when placed next to circular arcs representing the 24 in. and 6 in. pipes. The simulated hanger, which is detailed in lower sketch of Fig. 11, is seen to represent a reasonable model of the actual one to be tried in the 24 in. sewer. The main difference between these is that the actual hanger is 4 in. long in the flow direction whereas the simulated hangers tested were made from 1/4 in. thick aluminum. It was not too feasible to make these longer since some 140 were needed; it is assumed that this length difference had little effect on the flow since both designs had sharp leading edges thus causing flow separation at the front of each hanger.

The simulated hangers were slid along the 3/4 in. insert and they were held in place with a small nail driven through holes in the hanger and insert. As indicated in the sketch of Fig. 11 the hangers held the insert pipe about 0.83 in. away from the pipe wall, i. e., at an  $\epsilon$  of 0.737. Two different spacings in the flow direction, were tested, namely at 7 inch spacing and at an alternating 8 and 12 inch spacing. In order to conserve setup time these were tested simultaneously with the closer, uniform, spacing coming first and the staggered, larger, spacing following it along the insert. The change in spacing was after station 3 (see Fig. 4), i. e., about 50 ft. from the start of the insert. Pressure drop measurements were made between station 2-3 for the 7 in. spacing and 4-5 for the 8-12 in. spacing. In view of the extreme angularity of these hanger shapes, it is felt that their effects should have been well developed, or redeveloped for the 8-12 combination, easily in the  $72 D_{eq}$  and  $48 D_{eq}$  distances. Although the simulated hangers were always several inches from the piezometer rings used for the pressure drop measurements, the piezometer hole of each ring peripherally closest to the hangers was closed off to avoid any possible interference effects.

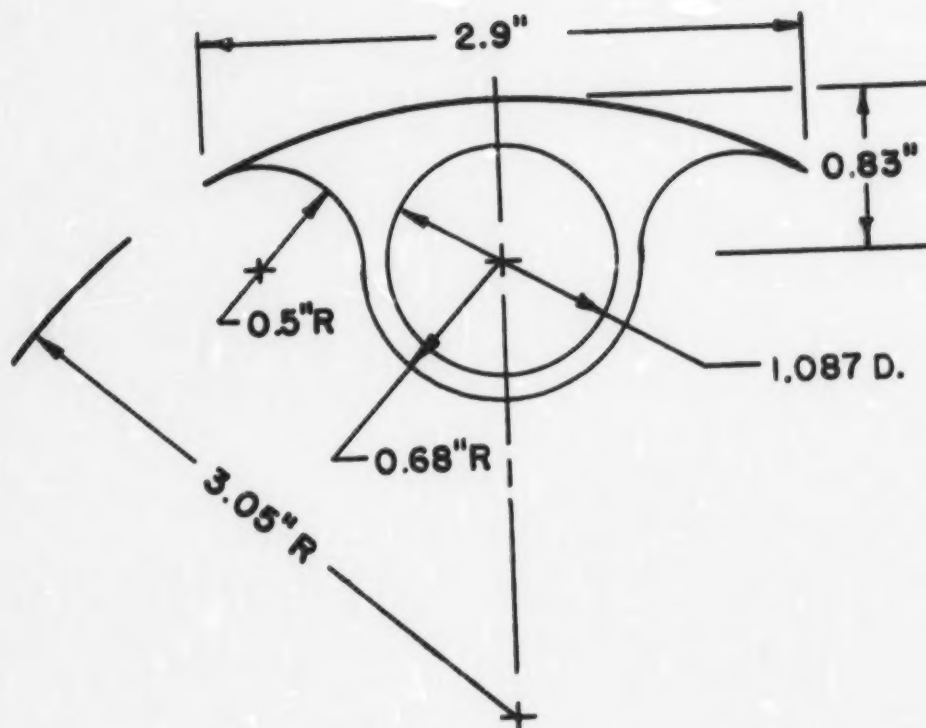
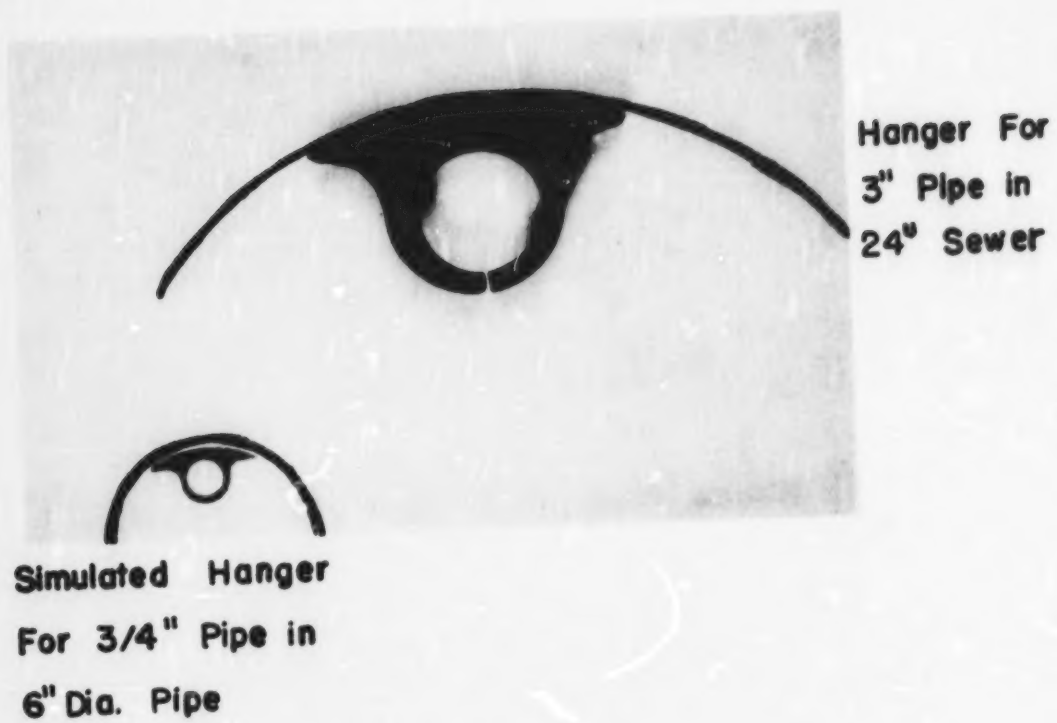


Fig. II Insert Hanger

The frictional pressure drop of the insert with simulated hangers was measured in similar fashion to the tests of the inserts already detailed. Resultant friction-factor variation with Reynolds number data is presented in Fig. 12 together with the reference results for the 6 in. pipe. In reducing the measurements the effect of the hangers on the cross sectional flow area (as volume per unit length) of the annulus was ignored, i. e.  $D_{eq}$  and area were taken to be the same as employed for the tests without simulated hangers. The data may thus be compared directly with that presented in Chapter III and is summarized in Appendix C.

Inspection of Fig. 12 indicates that, at the highest Reynolds number tested, the simulated hangers increased the friction factor to 2.73 and 2.41 times the smooth pipe values. The uniform 7 in. spacing had the larger friction. For the insert used ( $a = 5.79$ ) at  $\epsilon = 0.737$ , the predicted  $f$  value is  $0.95 f_{pipe}$ , so that the hangers have increased  $f$  by a factor of 2.86 and 2.54. At the lower Reynolds numbers the friction factors in both cases appear to become larger than expected from the convention friction formulations, even for rough surfaces. This is, perhaps, not overly surprising in view of some of the peculiar occurrences found in pipes with extreme type roughness. Friction factors of 0.045 to 0.05 are rather unusual and one questions the reasonableness of these results since the hangers were not large items. The frictional effects of slat-type artificial roughness has been summarized in a recent study.\* The effective sand-grain roughness of slats may be as much as 12 times the actual height if it is at the optimum spacing-height ratio. This is for a uniform peripheral distribution. Estimating the hangers to have an effective height of 0.37 in. and that their effect is in proportion to the amount of periphery occupied (3 in.) an effective sand-grain roughness of 0.46 in. is found. For this  $f$  would be 0.089, if fully rough flow obtains, which is hardly certain. The measured  $f$  values thus do seem possible.

The effect of these hangers on the annular pipe friction is thus seen to be considerable. Since appreciable added loss results from flow separation at the front edge of the hanger, it would be worthwhile to round this leading edge as much as possible. The need for care in designing the hangers is emphasized by noting that these increases in friction factor imply a reduced flow capacity to about sixty percent (actually to 0.57 and 0.61) of the bare sewer flows.

#### Insert Effects on Non-Full Flow of Sewer

A sewer only flows full on rare occasions. But when it flows full the occurrences

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\* A Study of Turbulent Flow in Rough Pipes, by J. M. Robertson, T. H. Burkhart and J. D. Martin, TAM Report 279, May 1965.

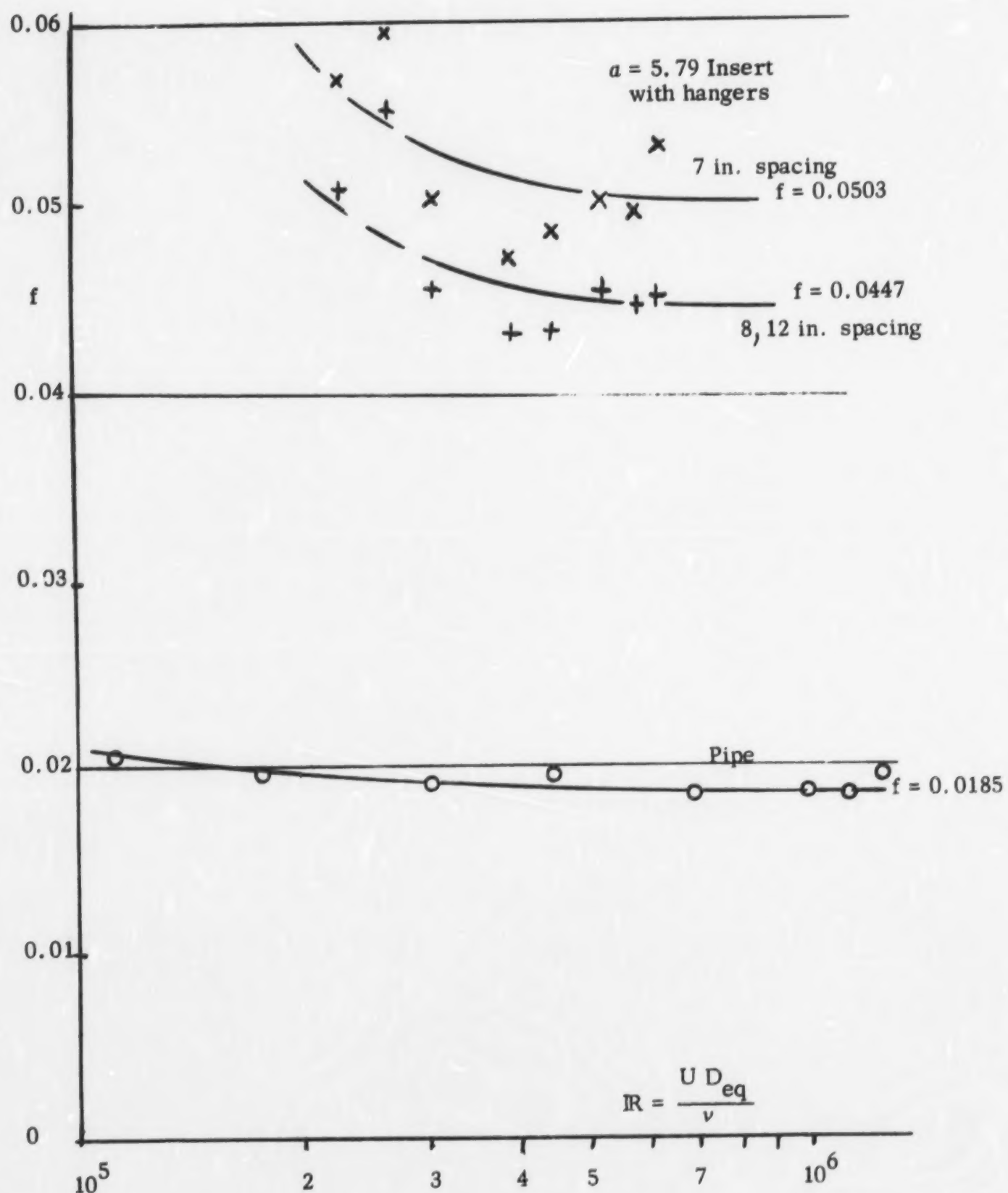


Fig. 12. Friction for  $a = 5.79$  Annulus with Hangers Compared with that for Pipe

are most significant. It is most important to know just how great the effect of the insert may be in this limiting case. Likewise it is desirable to have some indication of how the sewer capacity is affected for the near-full open-channel flows when the insert is in the free surface. Of course, in this region any disturbance may cause the free-surface to strike the crown of the sewer, and then full flow will occur.

The effect of an insert on the near-full flow of a sewer has been evaluated for the diameter-ratio case corresponding to the smaller insert studies, i. e., for an insert of diameter  $0.174 = 1/5.79$  of that of the circular sewer. As has been indicated in this case the full-flow capacity is reduced by only 4.5 percent; hanger effects are ignored in this analysis, although they could be included. Obviously, when the flow depth is less than  $0.826 = 1.000 - 0.174$  of the sewer diameter the capacity is unaffected. The capacity change due to the insert thus varies from zero at this flow depth to 4.5 percent at full depth. Since the change is so small, it might be adequate to prorate it in a simple fashion. However, a more refined method has been employed. The 4.5 percent change at full flow is due in part to area change -- 3 percent -- and in part to increased friction resulting from the increase in wetted surface. Therefore, it is assumed that a flow capacity change varying from zero to 3 percent, in accord with the area displaced by the insert, plus one of zero to 1.5 percent, in accord with the surface area of the insert which is wetted, must be applied to the flow capacity prediction for the partly full sewer.

It is commonly accepted that a circular conduit flowing at near full depth will carry more water than it will when full. The indication of such occurrences is shown in Fig. 13. Two of the indications presented there are on the assumption of constant friction coefficients; one of these is based on the conventional friction factor of Eq. 5 and the other on the common open-channel flow relation of Manning

$$(14) \quad Q = A R^{2/3} \frac{1.49}{n} S^{1/2}$$

where  $4R$  is  $D_{eq}$ . For a given sewer, then,  $Q$  is proportional to  $A R^{2/3}$ . In the case of the Darcy-Weisbach friction relation of Eq. 5, which is equally applicable to open-channel flow (with  $\Delta h/L = S$ , the slope), the flow  $Q$  is proportional to  $A R^{1/2}$  for a given sewer. The differences between the predictions of both of these relations on the variation in capacity of a circular conduit in the near-full condition are seen to be small. The Manning relation predicts a maximum flow of 1.08 times the full flow at a flow depth of 0.94 of the diameter, while the Darcy-Weisbach relation predicts a maximum flow of about 1.05 at about the same relative depth. The third type of prediction

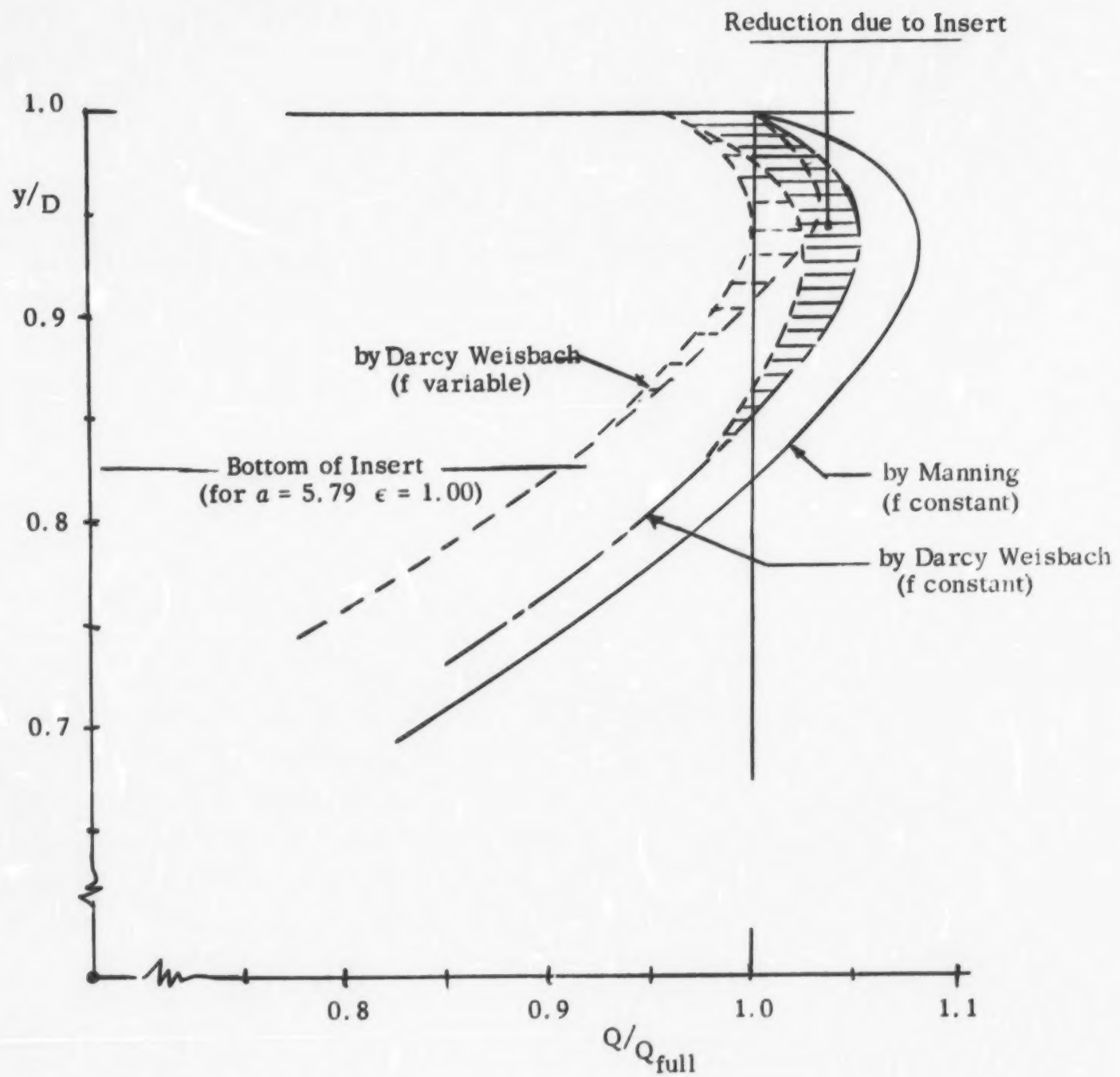


Fig. 13. Flow Capacity Variation at Near Full Condition With and Without Full Eccentric Insert at Crown.

indicated in Fig. 13 is based on indications of a variation in  $f$  with depth.\* This results in a more conservative indication of maximum discharge as about  $1.03 Q_{full}$ .

The correction for the influence of the inserted pipe, of diameter  $1/5.79$  that of the main conduit, has been applied to the  $f$ -type predictions, although it could probably just as well be applied to the Manning type. It is seen, from the presentation in Fig. 13, that the inserted pipe will reduce the maximum flow capacity to about 1.02 times that of the full flow capacity (without insert). A similar maximum capacity reduction is expected for the other case.

These considerations have been for the inserted pipe mounted against the crown of the sewer without consideration of the nature of the hangers. Should hangers of the type modeled in the experiments described in the preceeding section be employed, the reduction in flow capacity as the free surface reaches and engulfs the hangers and insert pipe will be quite considerable and roughly discontinuous in occurrence. For the specific geometry tested there will be no effect up to a depth of three-quarters of the diameter. At that point the flow rate (see Fig. 13) will be 0.78 to 0.91 of the full flow capacity of the sewer. Once the free surface strikes the lower edge of the hangers the friction factor will increase rapidly to approach the full-flow value of  $1/2.84$  or  $1/2.54$  times that of the bare sewer. As noted above this implies a flow reduction to about sixty percent of the capacity of the bare sewer.

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\* This is taken from ASCE Manual No. 37 (Fig. 35) "Design and Construction of Sanitary and Storm Sewers", 1960 (corrected 1961).

## V. SUMMATION

This study of turbulent friction in annuli has involved a thorough review of the literature on this subject and a number of measurements of the friction, and in particular that for the eccentric inserts.

### Conclusions

The general manner in which the friction factor, in turbulent flow through concentric annuli, varies with diameter ratio is indicated in Fig. 1. Despite considerable scatter between the results found by different investigators, it appears that the friction factor for annular conduits of diameter ratio near unity is about ten percent below that for the simple pipe, that it initially increases rapidly with diameter ratio to a value some ten percent above this reference at a ratio of about two and then slowly drops off towards unity as the diameter ratio further increases. Thus at a diameter ratio of six the friction factor is probably only about two percent above that for the simple pipe.

The effect of eccentricity is considerable. At maximum eccentricity (insert against inner wall of larger pipe) the friction factor is reduced to 0.64 of its concentric value, when the insert is nearly of the same diameter as the pipe. This reduction decreases as the relative size of the insert becomes smaller; thus the ratio is 0.82 when the insert diameter is one fifth that of the pipe. The mode of variation in the reduction in friction with eccentricities between zero (concentric case) and full is found although this is based on limited data. The general occurrences are indicated in Fig. 10.

The effect of inserts on flow capacity of a given conduit under a given pressure drop, or of a sewer on a given slope, is appreciably less than one expects from friction factor considerations. For the concentric position an insert, of diameter  $1/5.79$  of that of the pipe in which it is inserted, the flow capacity is reduced 12.7 percent. When positioned at full eccentricity, however, the capacity is reduced only 4.5 percent.

The use of hangers for the inserts, such as tested for this study, will have a considerable effect on the full flow capacity of the annular conduit. For the specific design tested the capacity was reduced by about forty percent. Prototype hangers should be fabricated with as little obstruction as possible to the flow area and with their leading edges fully rounded.

### Future Work

The number of cases (thus 5) which were studied in the experiments described is rather small and yet took quite a bit of effort. The possibility of clearing up the nature of the  $a$ ,  $\epsilon$ ,  $k/D$  and  $k_i/k_o$  effects on  $f$  via experiment thus seems remote. And yet

analytical studies, such as that of Deissler and Taylor (1955) and the simple extension of that outlined in Appendix B, offer one the opportunity to assess the manner in which these factors enter into the friction specification. Although there will still remain some uncertainties (to be resolved only by experiment) in the analytical prediction of turbulent friction, particularly in the nonaxisymmetric cases, such analysis certainly should indicate the pattern and trends of the occurrences. As a second phase of this general study of annular conduit friction flow, such turbulent flow analysis is in progress.

#### Acknowledgements

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## VI. LITERATURE ON ANNULAR CONDUIT FLOW

Review Summaries

H. L. Dryden, F. P. Murnaghan and H. Bateman, 1932, Hydrodynamics Bul. 84 National Res. Council, Wash. D. C., reprinted by Dover Publications, Ind., New York 1956, pp. 449-451.

Mentions annular pipes as a particularly severe test of hydraulic radius hypothesis since there\*  $r_h = R_o - R_i$ . Discusses tests by Becker (indication of critical IR of 2700 vs usual 2000), Atherton ( $f_{an} = 1.26 f_{cir}$  for turb. and 1.36 for lam case vs 1.43 to 1.49 ratio from laminar theory), Winkel ( $\Delta p$ , as per Blasius, prop. to  $vel^{1.75} r_h^{-1.25}$ ), and Lonsdale ( $\Delta p$  prop. to  $vel^{1.72} r_h^{-1.28}$  and crit.  $vel. = 4000 \sqrt{R_o / (R_o^2 - R_i^2)}$ ).

L. Schiller, 1932, "Die Stromung im geraden Rohr nicht kreisformigen Querschnitts; par. 1 Ringspaltformigen Querschnitt" (The Flow in Straight Pipe of Non-circular Cross Section: Annular Cross Section) in Wien-Harms Handbuch der Experimental Physik, vol IV-4, pp. 138-145.

A systematic review of theory and experiment. Under theory gives Becker's (1907) result for laminar flow\*\* (here corrected for an obvious typographical error)

$$\text{concentric} \quad \frac{\Delta p}{L} = 8 \mu U \frac{R_o^2 - R_i^2}{R_o^4 - R_i^4 - \frac{R_o^2 - R_i^2}{\log(R_o/R_i)}}^2$$

for very narrow ( $R_i \rightarrow R_o$ ) clearances, this is  $\frac{\Delta p}{L} \approx \frac{12 \mu U}{s^2}$

the eccentricity effects for narrow clearances is given by

$$\frac{\Delta p}{L} \approx \frac{12 \mu U}{s^2} \frac{1}{1 + \frac{3}{2} \frac{e^2}{s^2}}$$

where  $s = \text{clearance} = R_o - R_i$  and  $e = \text{eccentricity}$  (range 0 to  $s$ ). For  $e = s$ , i. e. inner pipe at wall of outer, the through flow is 2.5 times.

Calculates hydraulic radius on German basis (of 2 area/wetted perimeter), but here we translate that to the usual one (area/wetted perimeter) thus  $r_h = \frac{s}{2}$ , for

\* But the hydraulic radius is actually  $(R_o - R_i)/2$

\*\* See Lamb's Hydrodynamics, 6th Ed., p. 587.

Reynolds number uses  $s U \rho / \mu = R$ . Friction factor  $\psi = \frac{\Delta p}{L} \cdot \frac{s}{U^2/2}$  gets  $\psi = 24/R$  for narrow annuli (also p. 147, plane two-dim. channel).

The discussion of experiments covers the work of Becker, Winkel, Lonsdale, Schneckenberg and Lea and Tadros in some four pages.

- E. S. Davis, 1943, "Heat Transfer and Pressure Drop in Annuli" Trans. ASME, vol. 65, pp. 755-759.

Correlates friction data from Becker, Atherton, Schneckenberg, Kratz, MacIntire and Gould and Zerban. Finds a friction factor to vary as  $(R)^p \left( \frac{D_o}{D_o - D_i} \right)^b$  with  $p = 1$  in the laminar case and  $-0.2$  in the turbulent but  $b = 0$  in the laminar and  $-0.1$  in the turbulent. This latter business says that the turbulent flow in an annulus has a lower friction factor than for ordinary pipe; this correlation seems to agree with the experiments of Kratz et al, but most experiments indicate that  $f$  increases with diameter of inserted pipe.

- H. Richter, 1958, Rohrhydraulik (Pipe Hydraulics) 3rd Ed., Springer-Verlag, pp. 152-156.

For turbulent flow presents Lorenz's results as  $f = a + b/R^c$  ( $f$  and  $R$  based on  $d_{eq} = 4A/WP$ ) with  $a \approx 0.1$ ,  $b = 0.9$  to  $2.2$  as  $d_i$  increases and  $c = 0.4$  to  $0.56$ . For laminar says take  $f = \phi \frac{64}{R}$ ; Fig. 96 gives  $\phi$  vs  $R_o/R_i$  for laminar cases ( $\phi = 1.5$  for  $R_o/R_i = 1$  drops slowly (vs  $R_o/R_i$  on log scale) thus  $1.4$  at  $10$  to  $1.25$  at  $100$  to  $1.00$  at  $\infty$ ). Thus Eq. 263 gives\*

$$\phi = \frac{(R_o - R_i)^2 \ln R_o/R_i}{(R_o^2 + R_i^2) \ln R_o/R_i - (R_o^2 - R_i^2)}$$

- J. G. Knudsen and D. L. Katz, 1958 Fluid Dynamics and Heat Transfer McGraw-Hill

Book Company, pp. 91-98, 186-193 and 198-200. For laminar flow discuss analysis and experimental observations on velocity distribution for concentric case. For friction gives  $\phi$  vs  $D_o/D_i$  as varying from  $1.5$  for  $1$  to  $1.39$  for  $10$  to  $1.25$  for  $100$  (same equation as given by Richter).

For turbulent flow summarizes the velocity profile measurements of Rothfus and Knudsen and Datz and considers friction effects. For friction factor based on effective diameter suggests Davis's relation (which has been noted as probably wrong for the turbulent case) as well as a simple one ( $f/4 = 0.076/R^{0.25}$ ) independent of diameter ratio, since Davis's equation shows this effect to be small for  $D_o/D_i > 3$ . Briefly present Deissler and Taylor's calculation results for effect

\* This relation for  $\phi$  may be obtained from Becker's  $\Delta p$  relation given above.

of eccentricity on turbulent friction.

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$$IR \left[ \frac{a+1}{(a-1)^2} - \frac{a^2-1}{(a-1)^2 \ln a} \right]$$

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## APPENDIX A

Derivation of Eccentricity Factor for Laminar-Viscous  
Flow in Narrow Annuli

When the annulus is narrow, i. e., for  $a \approx 1$ , variations in the clearance between neighboring walls is small and the flow at any locale about the periphery of the pipe may be taken as that between two parallel plates spaced distance  $h$  apart. This is simply laminar plane-Poiseuille flow for which the mean flow velocity is

$$U = - \frac{h^2}{12\mu} \frac{dp}{dx}$$

where  $\mu$  = fluid viscosity

$\frac{dp}{dx}$  = axial pressure gradient

It is assumed that the pressure gradient is the same for all peripheral stations in the eccentric annulus.

The radial clearance between the surfaces is

$$h = \Delta r + \Delta c \cos \theta = \Delta r (1 + \epsilon \cos \theta)$$

where  $\Delta r = \frac{1}{2} (D_o - D_i)$

$\Delta c$  = offset of center of  $D_i$  from that of  $D_o$

$\theta$  = angular position about axis of conduit

$$\epsilon = \frac{2 \Delta c}{D_o - D_i} = \frac{\Delta c}{\Delta r}$$

The hydraulic radius is then  $\Delta r/2$ .

The flow rate down a narrow strip of annulus, of width  $r d\theta$  is

$dQ = U h r d\theta$   
where  $r = \frac{D_o + D_i}{4} = \frac{D_i}{2} + \frac{\Delta r}{2}$  is the average radius of the annulus. In the limit ( $D_o \rightarrow D_i$ ) considered here this is simply  $D_i/2 = D/2$ . On substituting the above fluid dynamic relations for  $U$  we find

$$\begin{aligned} dQ &= - \frac{h^2}{12\mu} \frac{dp}{dx} h \frac{D}{2} d\theta = \left( - \frac{D/2}{12\mu} \frac{dp}{dx} \right) h^3 d\theta \\ &= \left( - \frac{D}{24\mu} \frac{dp}{dx} \right) (\Delta r)^3 (1 + \epsilon \cos \theta)^3 d\theta \end{aligned}$$

The total flow down the annulus is found by integrating on  $\theta$  around the periphery, thus

$$Q = \left( -\frac{D}{24\mu} \frac{dp}{dx} \right) (\Delta r)^3 \int_0^{2\pi} (1 + \epsilon \cos \theta)^3 d\theta$$

or

$$\begin{aligned} \frac{Q}{\left( -\frac{D}{24\mu} \frac{dp}{dx} \right) (\Delta r)^3} &= \int_0^{2\pi} (1 + 3\epsilon \cos \theta + 3\epsilon^2 \cos^2 \theta + \epsilon^3 \cos^3 \theta) d\theta \\ &= 2\pi \left( 1 + \frac{3}{2} \epsilon^2 \right) \end{aligned}$$

The average flow velocity  $U_{avg} = Q/A$  in the annulus is then

$$U_{avg} = -\frac{1}{12\mu} \frac{dp}{dx} (\Delta r)^2 \left( 1 + \frac{3}{2} \epsilon^2 \right)$$

since the flow area is  $A = \pi D (\Delta r)$ .

In terms of  $f$  and  $IR$  based on  $U_{avg}$  and  $D_{eq} = 2(\Delta r)$  this result appears in the form

$$f IR = \frac{96}{1 + \frac{3}{2} \epsilon^2}$$

For verification we note that in the concentric case  $\epsilon = 0$  this reduces to

$$f IR = 96$$

as it should. Since  $IR$  is independent of  $\epsilon$ , the eccentricity factor  $E$  is thus

$$E = \frac{f_{ecc}}{f_{conc}} = \frac{96/(1 + 1.5 \epsilon^2)}{96} = \frac{1}{1 + 1.5 \epsilon^2}$$

as was indicated by Winkel in 1917. In the other limit  $\epsilon = 1$ , we have  $E = 0.4$  and  $f IR = 38.4$ .

## APPENDIX B

Estimate of Turbulent Friction for a Narrow Annulus  
Under Full Eccentricity  
by A. R. Nelson

The friction in a narrow annulus can be evaluated by dividing it into small two-dimensional channel elements of constant angular width  $\Delta\theta$ . In the fully eccentric case, the wall spacing of these elements varies from zero to twice the average clearance. If  $\Delta\theta$  is small, the change in clearance over the width is minimal, and an average clearance for the strip,  $h_s$ , can be determined. Since the head loss per unit length,  $\frac{\Delta p}{\gamma L}$ , must be the same for each of these channel elements, the following relation exists:

$$(A) \quad \frac{f_s U_s^2}{2h_s} = \text{constant}$$

where  $U_s$  = the average velocity in a strip

$h_s$  = the average wall spacing for a strip

By considering the flow to be incompressible and of constant viscosity throughout the annulus, it is shown that this head loss relation can be reduced to a form having the friction factor as a function of the clearance only. The frictional effects then are summed to evaluate the effective friction factor for an eccentric annulus.

The general equation for the local wall spacing of an annular strip is

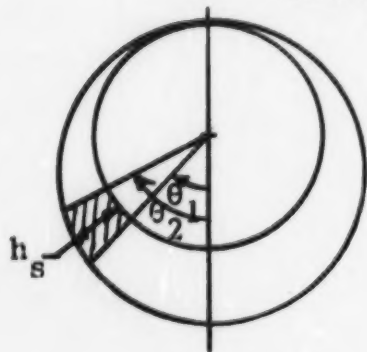
$$(B) \quad h_s = R_i \left[ \epsilon (a - 1) \cos \theta - 1 \right] + R_i \sqrt{a^2 - \epsilon^2 (a - 1)^2 \sin^2 \theta}$$

where  $R_i$  = radius of inner wall

$\epsilon$  = eccentricity

$a$  = diameter ratio

$\theta$  = reference angle (see sketch)



The area of a section of  $\Delta\theta = \theta_2 - \theta_1$  is found as follows

$$A = \int_{\theta_1}^{\theta_2} \int_{R_i}^{R_i + h_s} r \, dr \, d\theta$$

$$(C) \quad A \cong \frac{1}{2} \left[ \frac{2h_s R_i + h_s^2}{2} \Big|_{\theta_1} + \frac{2h_s R_i + h_s^2}{2} \Big|_{\theta_2} \right] \Delta \theta$$

The friction at any peripheral location in narrow annuli can be evaluated from the relation for wide rectangular ducts as given previously.

$$(II) \quad \frac{1}{\sqrt{f}} = 2.03 \log_{10} (R\sqrt{f}) - 0.47$$

For ease of analysis, this equation was changed to a direct relation for  $f$  over the range of interest by plotting the friction factor as a function of a Reynolds number. A simpler equation approximating the general relation for the range  $R = 10^4$  to  $10^6$ , is

$$(D) \quad f = 0.167 R^{-0.202}$$

The variations in friction factor can now be related to the clearance via the fore-mentioned friction equations. By assuming that the same head loss occurs for each narrow strip, one has

$$(E) \quad \frac{f}{f_0} = \left( \frac{U_0}{U} \right)^2 \frac{h}{h_0}$$

In this analysis all quantities are expressed with respect to the corresponding values at the location of maximum width, i. e.  $\theta = 0$ , as indicated by the subscript 0. The local velocity ratio, as expressed in terms of the Reynolds number (with  $D_{eq} = 2h$ ), is given by

$$(F) \quad \frac{U}{U_0} = \frac{R}{R_0} \frac{h_0}{h}$$

On substitution of this relation into the general friction relation (Eq. 5), one obtains

$$(G) \quad \frac{f}{f_0} = \left( \frac{R_0}{R} \right)^2 \left( \frac{h}{h_0} \right)^3$$

To obtain the  $f$  vs  $h/h_0$  relation, the value of  $R_0$  will be arbitrarily chosen as  $1 \times 10^6$ . From Eq. (D), the corresponding value of  $f_0$  is 0.0102.

Solving Eq. (D) for  $R = R(f)$  and inserting this relation into Eq. (G), the resulting relation between  $f$  and  $h$  is

$$(H) \quad \frac{f}{0.0102} = \left( \frac{10^6}{\left( \frac{1.67}{f} \right)^{4.95}} \right)^2 \left( \frac{h}{h_0} \right)^3$$

The desired result of  $f$  vs  $h/h_0$  is obtained by indirect solution and plotting of this equation. Then with the ratio  $\frac{h}{h_0}$  given for each peripheral section and the calculated friction factor, the velocity ratio for any station  $s$  is

$$(I) \quad \frac{U_s}{U_0} = \left( \frac{f_0}{f_s} \frac{h_s}{h_0} \right)^{1/2}$$

The average flow velocity  $U_{av}$  and clearances are then found by summing over all the strips.

The particular analysis described here employed  $a = 1.01$ ,  $\epsilon = 1.0$  and  $\Delta\theta = \frac{\pi}{10}$ . The average flow velocity and clearance ratios are

$$\frac{U_{av}}{U_0} = \frac{\sum_{s=1}^{20} U_s A_s}{U_0 \sum_{s=1}^{20} A_s} = 0.810$$

$$\frac{h_{av}}{h_0} = \sum_{s=1}^{20} \frac{h_s}{h_0} = 0.5024$$

Again, for constant head loss throughout the entire annulus

$$f_{av} = f_0 \left( \frac{U_0}{U_{av}} \right)^2 \left( \frac{h_{av}}{h_0} \right) = 0.00781$$

since  $f_0 = 0.0102$ ,

The needed reference concentric friction factor for the narrow annulus can be found from the Reynolds number which is the average Reynolds number based on  $U_{av}$  and  $h_{av}$ . From Eq. (F), the resulting  $Re_{av}$  is  $4.17 \times 10^5$ . The corresponding friction factor, as obtained from Eq. (D), is 0.0122. Thus the ratio of  $\frac{f_{ecc}}{f_{conc}}$  for a narrow annulus in the fully eccentric condition is

$$\frac{f_{ecc}}{f_{conc}} = \frac{0.00781}{0.0122} = 0.640$$

This prediction has been indicated in Fig. 2 where it is seen to be consistent with the indications from experiment.

## APPENDIX C

## Test Data for Pipe and Annuli

| Q<br>cfs                                           | Velocity<br>fps | $\Delta h_{35}$<br>ft (H <sub>2</sub> O) | Man -<br>ometer         | IR                  | f       |
|----------------------------------------------------|-----------------|------------------------------------------|-------------------------|---------------------|---------|
| a. 6 in. Pipe                                      |                 |                                          |                         |                     |         |
| 4.85                                               | 23.9            | 13.63                                    | Mercury-Water           | $12.15 \times 10^5$ | 0.01988 |
| 4.555                                              | 22.45           | 11.21                                    |                         | 11.42               | .01852  |
| 3.844                                              | 18.94           | 8.04                                     |                         | 9.64                | .01863  |
| 2.750                                              | 13.55           | 4.08                                     |                         | 6.88                | .01852  |
| 1.720                                              | 8.47            | 1.682                                    |                         | 4.31                | .01955  |
| 1.720                                              | 8.47            | 1.685                                    | CCl <sub>4</sub> -Water | 4.31                | .01958  |
| 1.177                                              | 5.80            | 0.777                                    |                         | 2.955               | .01910  |
| 0.717                                              | 3.53            | 0.293                                    |                         | 1.797               | .01961  |
| 0.440                                              | 2.166           | 0.1158                                   |                         | 1.103               | .02049  |
| b. 6 in. Pipe with 3/4 in. Concentric Insert       |                 |                                          |                         |                     |         |
| 4.43                                               | 22.53           | 13.81                                    | Hg-Water                | $9.46 \times 10^5$  | 0.01872 |
| 3.65                                               | 18.56           | 9.39                                     |                         | 7.8                 | .0187   |
| 2.80                                               | 14.23           | 5.63                                     |                         | 5.98                | .0191   |
| 1.853                                              | 9.42            | 2.422                                    |                         | 3.96                | .0188   |
| 1.350                                              | 6.86            | 1.370                                    |                         | 2.87                | .02004  |
| 1.014                                              | 5.16            | 0.792                                    | CCl <sub>4</sub> -Water | 2.17                | .02047  |
| 0.604                                              | 3.07            | 0.260                                    |                         | 1.29                | .0190   |
| 0.430                                              | 2.18            | 0.149                                    |                         | 0.915               | .0214   |
| c. 6 in. Pipe with 3/4 in. Eccentric (full) Insert |                 |                                          |                         |                     |         |
| 4.66                                               | 23.77           | 12.82                                    | Hg-Water                | $9.65 \times 10^5$  | 0.01564 |
| 3.805                                              | 19.35           | 8.41                                     |                         | 8.2                 | 0.541   |
| 2.80                                               | 14.23           | 4.67                                     |                         | 6.05                | .01586  |
| 1.863                                              | 9.48            | 2.204                                    |                         | 4.02                | .01690  |
| 1.863                                              | 9.48            | 2.170                                    |                         | 4.02                | .01663  |
| 1.320                                              | 6.71            | 1.055                                    | CCl <sub>4</sub> -Water | 2.85                | .01612  |
| 0.975                                              | 4.96            | 0.568                                    |                         | 2.105               | .01528  |
| 0.555                                              | 2.82            | 0.1786                                   |                         | 1.20                | .01543  |
| 0.449                                              | 2.282           | 0.1162                                   |                         | 0.97                | .0153   |

| Q<br>cfs                                                | Velocity<br>fps | $\Delta h_{35}$<br>ft (H <sub>2</sub> O) | Man-<br>ometer                                 | IR                 | f       |
|---------------------------------------------------------|-----------------|------------------------------------------|------------------------------------------------|--------------------|---------|
| d. 6 in. Pipe with 1-1/2 in. Concentric Insert          |                 |                                          |                                                |                    |         |
| 3.898                                                   | 21.35           | 15.60                                    | Hg-Water                                       | $7.52 \times 10^5$ | 0.01965 |
| 3.002                                                   | 16.39           | 9.27                                     |                                                | 5.70               | .01979  |
| 2.321                                                   | 12.66           | 5.14                                     |                                                | 4.40               | .01839  |
| 1.250                                                   | 6.82            | 1.775                                    |                                                | 2.37               | .02185  |
| 1.250                                                   | 6.82            | 1.76                                     |                                                | 2.37               | .02168  |
| 1.080                                                   | 5.89            | 1.366                                    | CCl <sub>4</sub> -Water                        | 2.05               | .02262  |
| 0.797                                                   | 4.34            | 0.752                                    |                                                | 1.51               | .02290  |
| 0.530                                                   | 2.89            | 0.380                                    |                                                | 1.005              | .02608  |
| 0.306                                                   | 1.67            | 0.162                                    |                                                | 0.581              | .03325  |
| e. 6 in. Pipe with 1-1/2 in. Insert at 2/3 Eccentricity |                 |                                          |                                                |                    |         |
| 3.820                                                   | 20.82           | 13.70                                    | Hg-Water                                       | $7.34 \times 10^5$ | 0.01809 |
| 3.068                                                   | 16.73           | 9.14                                     |                                                | 5.89               | .01870  |
| 2.484                                                   | 13.57           | 6.06                                     |                                                | 4.78               | .01889  |
| 1.573                                                   | 8.59            | 2.56                                     |                                                | 3.02               | .01990  |
| 1.614                                                   | 8.80            | 2.525                                    |                                                | 3.10               | .01870  |
| 1.222                                                   | 6.67            | 1.578                                    | CCl <sub>4</sub> -Water                        | 2.34               | .01968  |
| 1.007                                                   | 5.48            | 1.054                                    |                                                | 1.93               | .02012  |
| 0.782                                                   | 4.27            | 0.671                                    |                                                | 1.50               | .02107  |
| 0.453                                                   | 2.47            | 0.302                                    |                                                | 0.87               | .02232  |
| f. 6 in. Pipe with 1-1/2 Insert at Full Eccentricity    |                 |                                          |                                                |                    |         |
| 3.945                                                   | 21.53           | 12.21                                    | Hg-Water<br>CCl <sub>4</sub> -H <sub>2</sub> O | $7.58 \times 10^5$ | 0.01511 |
| 3.359                                                   | 18.30           | 8.90                                     |                                                | 6.44               | .01522  |
| 2.897                                                   | 15.80           | 6.75                                     |                                                | 5.56               | .01549  |
| 1.979                                                   | 10.79           | 3.244                                    |                                                | 3.80               | .01599  |
| 1.369                                                   | 7.46            | 1.558                                    |                                                | 2.63               | .01602  |
| 1.090                                                   | 5.95            | 1.039                                    |                                                | 2.096              | .01680  |
| 0.756                                                   | 4.13            | 0.519                                    |                                                | 1.45               | .01740  |

| Q<br>cfs | Velocity<br>fps | IR | Hangers at 7 in.                         |                |   | Hangers 8", 12"                          |                |   |
|----------|-----------------|----|------------------------------------------|----------------|---|------------------------------------------|----------------|---|
|          |                 |    | $\Delta h_{23}$<br>ft (H <sub>2</sub> O) | Mano-<br>meter | f | $\Delta h_{45}$<br>ft (H <sub>2</sub> O) | Mano-<br>meter | f |

g. 6 in. Pipe with 3/4 in. Insert on Hanger

|      |       |                    |       |                        |        |       |                        |        |
|------|-------|--------------------|-------|------------------------|--------|-------|------------------------|--------|
| 3.24 | 10.04 | $6.21 \times 10^5$ | 10.16 |                        | 0.0533 | 8.27  |                        | 0.0451 |
| 2.94 | 15.02 | 5.82               | 8.30  |                        | .0497  | 7.22  |                        | .0449  |
| 2.69 | 13.74 | 5.32               | 7.01  | Hg-Water               | .0502  | 6.10  | Hg-Water               | .0454  |
| 2.31 | 11.80 | 4.57               | 5.01  |                        | .0487  | 4.31  |                        | .0434  |
| 1.97 | 10.17 | 3.94               | 3.61  |                        | .0472  | 3.18  |                        | .0432  |
| 1.55 | 7.94  | 3.07               | 2.34  |                        | .0504  | 2.05  |                        | .0456  |
| 1.36 | 6.94  | 2.68               | 2.10  | CCl <sub>4</sub> Water | .0593  | 1.90  | CCl <sub>4</sub> Water | .0553  |
| 1.15 | 5.90  | 2.28               | 1.46  |                        | .0569  | 1.264 |                        | .0510  |

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